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THE STABILITY OF PLANETS IN THE ALPHA CENTAURI SYSTEM

PAUL A. WIEGERT[1]
Department of Astronomy, University of Toronto, 60 St. George Street, Toronto, Ontario, M5S 8H8, Canada
Electronic mail: wiegert@yorku.ca

MATT J. HOLMAN
Canadian Institute for Theoretical Astrophysics, University of Toronto, 60 St. George Street, Toronto, Ontario M5S 3H8, Canada
Electronic mail: holman@cita.utoronto.ca

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ABSTRACT

This paper investigates the long-term orbital stability of small bodies near the central binary of the Alpha Centauri system. Test particles on circular orbits are integrated in the field of this binary for 32 000 binary periods or approximately 2.5 Myr. In the region exterior to the binary, particles with semimajor axes less than roughly three times the binary's semimajor axis a_b are unstable. Inside the binary, particles are unstable if further than 4 AU from either star, with stability closer in a strong function of inclination on the time scale of these integrations: orbits inclined near 90° are unstable in as close as 0.23 AU from either star. We also present charts of the predicted stable regions as projected against the plane of the sky to aid direct imaging searches. © *1997 American Astronomical Society.* [S0004-6256(97)02904-X]

1. INTRODUCTION

Though the formation of multiple star systems is possibly quite different from that of single stars like our Sun, it is plausible that multiple stars also host planetary systems. If this is the case, then the high frequency of binary and multiple systems implies that such planetary systems have been created in large numbers in our Galaxy. Recently, the question of whether planets might persist for long periods within such systems was answered empirically, with the discovery of extrasolar planets in 16 Cyg B, τ Boo, and 55 Cnc (Cochran *et al.* 1996; Butler *et al.* 1996). But the conditions under which such planets are stable are not yet understood.

Alpha Centauri, a triple system with two of the stars forming a close binary (semimajor axis 23 AU) and a third orbiting this pair at a much greater distance (12 000 AU), is extraordinary only in its proximity to the Sun (1.3 pc). For this reason, it is a prime place to prospect for planets, and a logical starting point for our theoretical investigations of the stability of planetary orbits in multiple systems.

Stability considerations can constrain the locations where planets are likely to exist. As direct imaging and astrometric techniques are most suited to detecting planets on large orbits, while spectroscopic methods are better suited to small orbits, an understanding of long-term stability in binary systems can increase the efficiency of searches for extra-solar planets. We seek regions of phase space where test particles (planets) could remain for times on the order of the ages of the stars. More precisely, we will determine those regions in which planets cannot be stable on such time scales. Our integrations follow test particles for only a few million years, and thus cannot assure stability over the α Cen system's probable 5 billion year age (Noels *et al.* 1991). However, even such relatively short integrations are sufficient to identify large regions in which single planets are unstable, and thus cannot exist today.

We adopt a simple, empirical, observationally motivated criterion for stability. The term ''stable'' will be applied to test particles whose time-averaged semimajor axis does not vary from its initial value by more than 5% over the whole integration, the remainder being termed ''unstable.'' Thus our definition of stability excludes planets which remain bound to the binary, but migrate to larger or smaller orbits, encompassing only such planets as remain near their initial orbits. It should be noted that, as all the dynamical processes in the model are time reversible, a particle which moves away from its original semimajor axis could easily return at a future time. The deviation in semimajor axis is only a measure of the available phase space.

We also compute Lyapunov times, which are measures of the rate of exponential divergence of nearby orbits and, hence, of chaos. Roughly speaking, smaller Lyapunov times are associated with predictability over shorter time intervals; an infinite Lyapunov time indicates the absence of chaos in the system.

2. METHOD AND MODELS

The numerical integrations in this paper used the symplectic mapping for the N-body problem described by Wisdom & Holman (1991). This technique is typically an order of magnitude faster than conventional integration methods and has the additional advantage of showing no spurious dissipation other than that introduced by roundoff error. Lyapunov times were computed by evolving a tangent vector

[1]Present address: Department of Physics and Astronomy, York University, 4700 Keele Street, North York, Ontario, M3J 1P3, Canada.

TABLE 1. Physical characteristics of α Cen A and B, including their mass, spectral type, apparent and absolute visual magnitude and luminosity (Kamper & Wesselink 1978; Hoffleit & Jaschek 1982; Lang 1992).

Star	Mass ($\mathcal{M}_\odot$)	MK class	V	M_V	L ($L_\odot$)
α Cen A	1.1	G2 V	−0.01	4.37	1.6
α Cen B	0.91	K1 V	1.33	5.71	0.45

associated with each test particle during the orbit calculations (Mikkola & Innanen 1994). This procedure has two advantages over the common approach of measuring the Lyapunov time by evolving two nearby trajectories. First, using the tangent vector avoids the saturation and renormalisation problems that accompany the two-trajectory technique. Second, the variational method is faster because the most expensive calculations required (the distances between the test particle and planets) do not need to be computed twice.

We approach the problem with a simple model which captures the overall dynamics. We ignore the distant third star, α Cen C (Proxima), as it appears likely that it is not bound to the central binary (Anosova *et al.* 1994), and because the perturbations it could inflict were it bound are likely small. The orbit of the central pair is thus taken to be a fixed Kepler ellipse. The semimajor axis of the central binary a_b is 23.4 AU, its eccentricity is 0.52 and the inclination of its orbit to the plane of the sky is 79° (Worley & Heintz 1983). The primary (α Cen A) has a mass of 1.1 $\mathcal{M}_\odot$; the secondary (α Cen B) has a mass of 0.91 $\mathcal{M}_\odot$ (Kamper & Wesselink 1978). Their physical properties are outlined in Table 1. In the field of this binary we integrate a battery of massless test particles representing low-mass planets. As these particles do not interact with one another, this paper does not address the stability of multiple planet systems.

Particles are terminated when they move to escape orbits or have close encounters with one of the stars. The algorithm is not optimzed to handle these efficiently and, in any case, close encounters indicate instability against either dynamical loss or tidal disruption. A close encounter is defined (arbitrarily) to be a passage within 5.9 AU ($0.25a_b$) of the secondary, or (for reasons of numerical accuracy) within 5 stellar radii ($0.001a_b$) of the primary. The exception to this rule occurs is the case of planetary orbits centered on the α Cen B, in which case the close encounter criteria are reversed.

3. INITIAL CONDITIONS

Test particles are initially placed in circular orbits in three separate regions: near the primary and near the secondary (the "inner" regions) and outside the binary's orbit (the "outer" region). As the central binary has minimum and maximum separations of 11.2 and 35.6 AU, particles on circular orbits around one of the stars and which have semimajor axes in this range suffer close encounters with the other star, and are unlikely to be stable. The interior region centered on α Cen A extends from 0.23 to 11.7 AU (0.01 to 0.5 times the binary semimajor axis a_b). Given the similarity of the stellar masses, the simulations of the region around α Cen B were performed at lower resolution and only at distances from 1.2 to 4.7 AU (0.05 to 0.2 a_b). The exterior region is centered on the barycenter, and spans 35 to 117 AU ($1.5a_b$ to $5a_b$).

Note that the mass fraction in the secondary (0.45) exceeds the maximum value (~0.005 for a binary eccentricity of 0.52, Danby 1964) at which the L_4 and L_5 Lagrange points are linearly stable, so no particles are expected to survive there.

The integration is started with the perturber at apastron, and on the opposite side of the primary from the particles. The particles are initially in the plane of the binary, but have a range of inclinations.

Thirteen different inclination values were examined, ranging from 0° to 180° in 15° increments. All particles were started on circular orbits, relative to the nearest star in the inner shells, and relative to the barycenter in the outer one. Particles were distributed evenly in initial semimajor axis a: 50 particles around the primary and 16 around the secondary (at 0.23 AU or $0.01a_b$ particle separation) for each value of the inclination, and 36 particles (at 2.3 AU or $0.1a_b$ particle separation) for each value of the inclination in the outer region, for a total of 1326 particles in all regions. The integration proceeded for 32 000 binary periods, approximately 2.5 Myr of simulated time.

The time step used was 3×10^{-3} of the binary period in the outer region; in the inner regions, the step size was 10^{-4} for semimajor axes from 2.34 to 11.7 AU, and 3×10^{-5} for semimajor axes less than and including 2.34 AU. These step sizes translate into 33 steps per particle orbit at 0.23 AU, 360 per orbit at 2.5 AU, and 610 per orbit at 35 AU. We have confirmed that these step sizes are small enough by comparing them to the results of test cases with small step sizes. Also, we have checked a number of test cases with a standard Bulirsch–Stoer numerical integrator.

When integrating particle orbits in the inner regions, the mass of the noncentral star was grown adiabatically over 500 binary periods in order to eliminate transients in the particles' motions which would not be present in a mature planetary system. The time span of 500 binary periods (roughly 40 000 years) is comparable to the precession period of test particle orbits with semimajor axes as small as 1.2 AU ($0.05a_b$) around the primary. The adiabatic growth procedure was found to have little effect on the final results, and was omitted in the calculations of the outer region.

4. SIMULATIONS

The regions near both α Cen A and B prove to be largely unstable over the integration time scale (Figs. 1 and 2). Each cell of these figures represents one of the test particles' initial conditions. A white cell indicates a particle that was ejected or had a close encounter. Two other colors indicate those particles that survived for the entire simulation: those whose time-averaged semimajor axis deviated from its initial value by less than 5% are indicated in black, those which deviated by more than 5% are shown in grey. Note that a near-constant semimajor axis does not imply a consistently near-circular orbit, and many long-lived particles do reach large

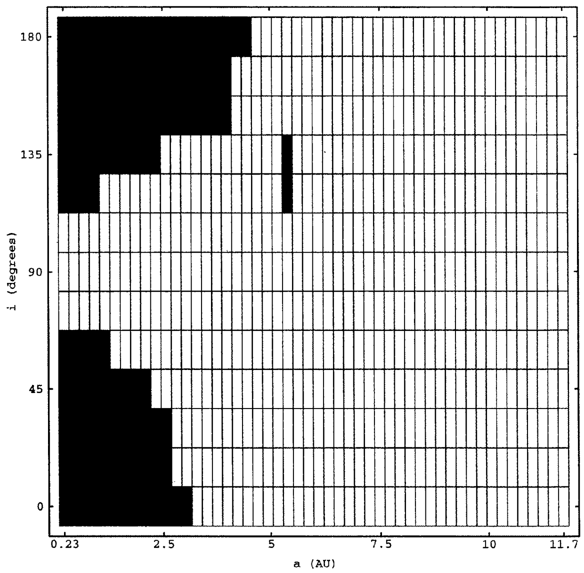

FIG. 1. The change in semimajor axis of test particles in the region near α Cen A, on a grid of inclination i and semimajor axis a. A white cell indicates a particle that was ejected or had a close encounter with one of the stars. Particles which survived the whole integration time, but whose average semimajor axis differs from its initial value by more than 5% are indicated by a grey cell, while a difference of less than 5% is indicated by a black cell.

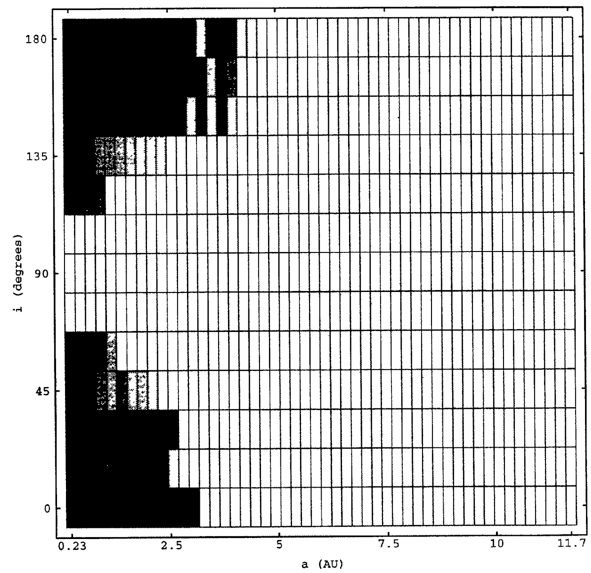

FIG. 3. The Lyapunov time of test particles in the region near α Cen A, on a grid of inclination i and semimajor axis a. Black indicates the longest detectable Lyapunov times (approximately 1200 binary periods), shading to white, the lowest, at or near zero. Particles which were ejected, suffer close encounters with one of the stars or which moved by more than 50% of their initial semimajor axis are grouped with the lowest Lyapunov times (white).

eccentricities, for reasons discussed in Sec. 5.

The regions where particles are longest-lived are close to each of the stars as might be expected, but with a wide gap at inclinations between 60° and 120°. Retrograde orbits survive out to larger radii than prograde ones, as might be expected from the shorter encounter times suffered by retrograde orbits and from studies of distant outer planet satellites (Hénon 1970).

There are only a few grey particles in Figs. 1 and 2, all of which either migrate out to large orbits outside the binary ($a \gtrsim 100$ AU), or move slightly inwards, typically by 10% or so. Overall, the a-i plane is divided fairly cleanly into two

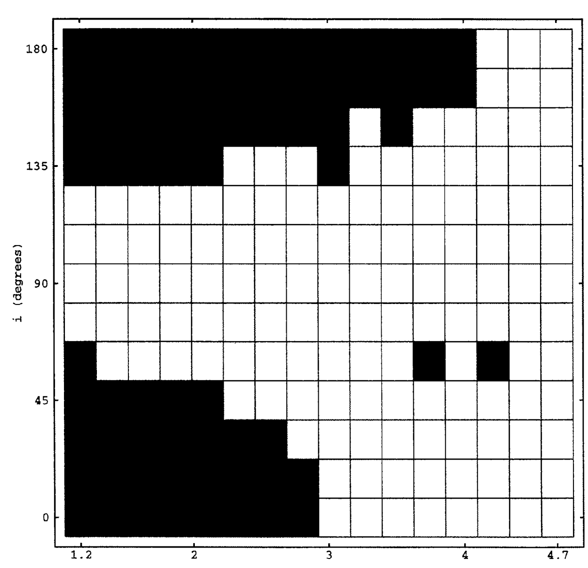

FIG. 2. The change in semimajor axis of test particles in the region near α Cen B, on a grid of inclination i and semimajor axis a. The color scheme is the same as for Fig. 1.

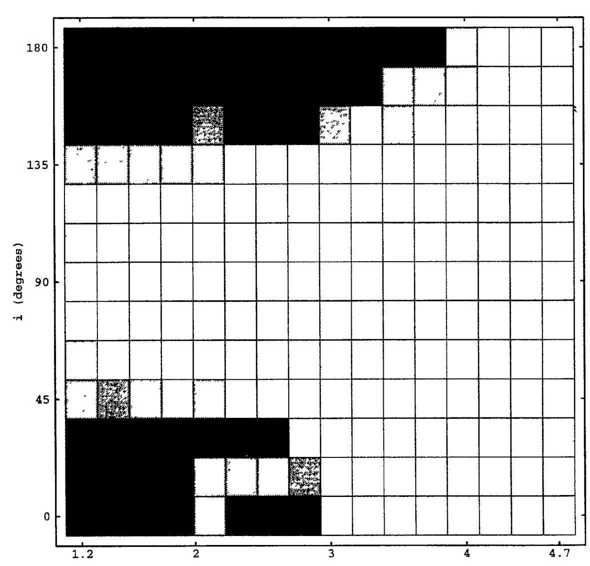

FIG. 4. The Lyapunov time of test particles in the region near α Cen B, on a grid of inclination i and semimajor axis a. The color scheme is the same as for Fig. 3.

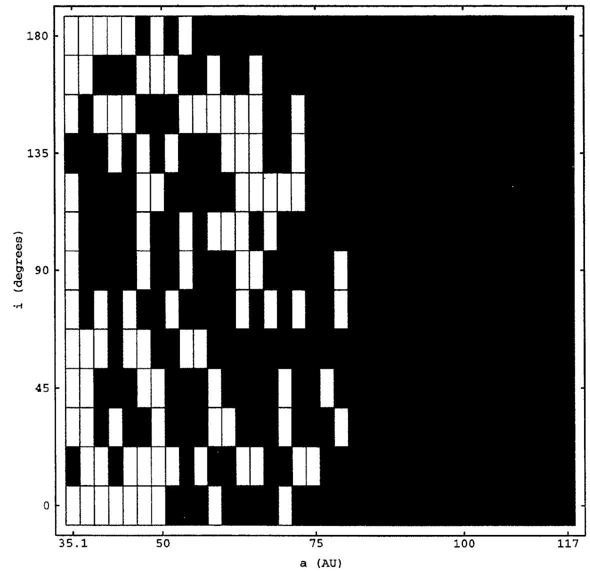

FIG. 5. The change in semimajor axis of test particles in the outer region of the α Cen binary. The shadings are the same as in Fig. 1.

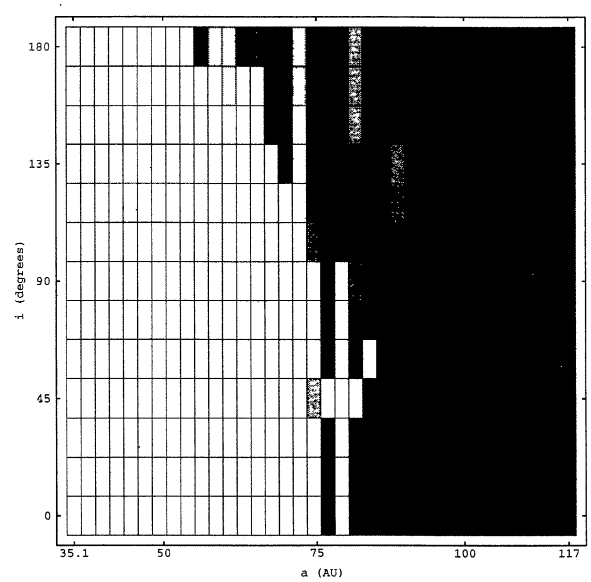

FIG. 6. The Lyapunov time of test particles in the outer region of the α Cen binary. The shadings are the same as in Fig. 3, except that the maximum Lyapunov time is now roughly 2800 binary periods.

parts, one stable and one unstable on million year time scales.

Plots showing Lyapunov times for the regions near α Cen A and B appear in Figs. 3 and 4, and show the same general stable/unstable division as Figs. 1 and 2. There are, however, particles with Lyapunov times below the maximum detectable level (which is about 1200 binary orbital periods, or 0.1 Myr in the regions near both stars), but which have not moved significantly from their initial positions. As the time scale for large qualitative changes in test orbits has been observed to be much longer than the Lyapunov times in some cases (Lecar *et al.* 1992), such particles may move away from their initial positions over longer time scales.

The simulation results for the outer region are shown in Figs. 5 and 6. The region is again divided into stable and unstable regions. Particles within roughly 70 AU ($3a_b$) are for the most part unstable with the stable region reaching further inwards for retrograde orbits. Orbits outside 70 AU typically survive for the length of the integration.

There are many grey cells in Fig. 5, indicating particles which have survived the integration but whose semimajor axes wander significantly from their initial values. Almost all of these particles move to larger orbits well outside the binary. Only one moves to a more tightly bound orbit, and a few along the stability edge remain within 50% of their initial semimajor axis, but these will presumably move away on longer time scales.

The behavior of the Lyapunov times in the outer region, displayed in Fig. 6, is generally consistent with Fig. 5. Particles which show only small changes in semimajor axis generally have longer Lyapunov times, except for a few isolated particles. It is unclear if these exceptions indicate isolated pockets of chaotic behavior associated with narrow resonances, or possibly chaos on time scales of order or slightly larger than the maximum detectable Lyapunov time, and which are just at the edge of detectability in these simulations. The maximum detectable Lyapunov time is slightly longer outside than inside, at roughly 2800 orbits or 0.2 Myr.

In order to investigate the stability of planets over even longer time scales, all particles around the primary between 1.2 and 3.5 AU and at zero inclination were run 50 times longer (1.6 million binary periods or 130 Myr). Particles inside and including that at 2.34 AU were stable, with no signs of chaos on time scales less than the maximum detectable Lyapunov time (roughly 50 000 binary periods or 4 Myr) while the particles outside 2.34 AU were all ejected or suffered close encounters. Comparison of this result with those in Figs. 1 and 3 indicates that the inner edge of the ``stable'' region may be eroded somewhat as integration times are extended, though the time scale for this effect and whether or not it will reach arbitrarily far inwards is unclear.

5. QUESTIONS AND COMMENTS

Although these integrations cannot assure the stability of planets on time scales greater than about a million years, they do identify important unstable regions. The zone in which planets cannot have survived since the formation of the α Cen system extends from roughly 3 AU from each star to 75 AU (0.15 to $3.2a_b$) from the barycenter for orbits which lie in the binary's orbital plane. Retrograde orbits may be stable inside 4 AU and outside 66 AU. Orbits lying perpendicular to the plane are unstable in as close as 0.23 AU from the primary, though smaller stable orbits are not excluded by our studies.

Here we offer a brief heuristic explanation of this gap at high inclinations in the interior region, following the analysis of Kozai (1979) and Quinn *et al.* (1990). Considering only test particles orbiting near one of the stars, we replace the perturbing secondary mass by a circular ring of the same mass. The canonical momenta of the test particle orbit are

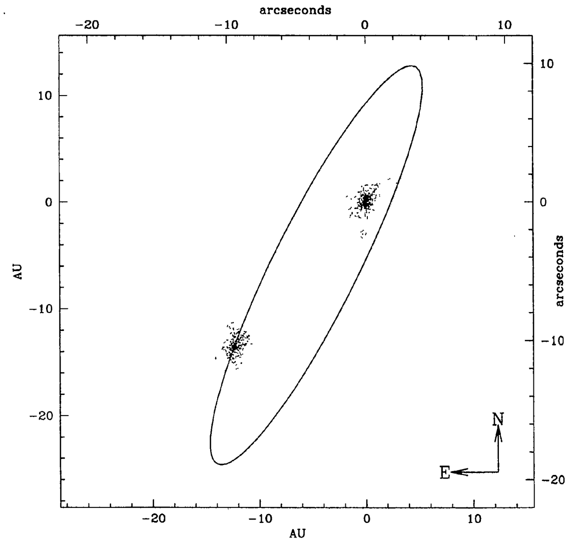

FIG. 7. The projected density of stable interior planets around the α Cen binary. The orbit of the secondary is based on Worley & Heintz (1983), and the secondary's position is indicated for the epoch 2000.0. The axes are measured in arcseconds.

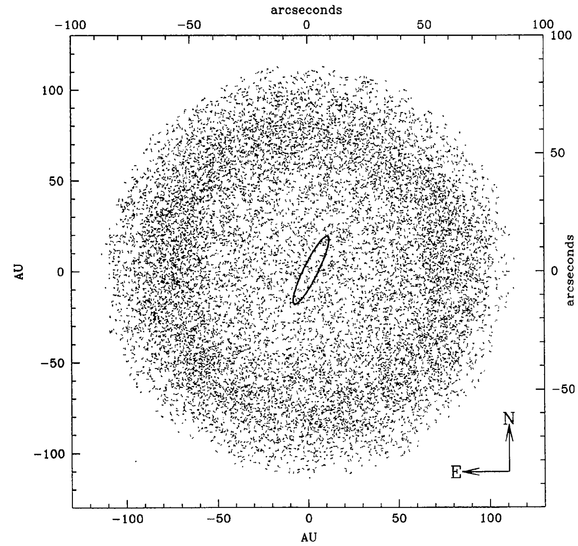

FIG. 8. The projected density of stable exterior planets with semimajor axes less than or equal to 117 AU around the α Cen binary. The position of the outer cloud boundary is an artifact resulting from our choice of 117 AU (5 a_b) as the maximum test particle semimajor axis.

$L=(\mu a)^{1/2}$, $J=[\mu a(1-e^2)]^{1/2}$, and $J_z=[\mu a(1-e^2)]^{1/2}\cos i$, where $\mu=G\mathscr{M}_0$, $\mathscr{M}_0$ is the mass of the primary, a is the particle's semimajor axis, e is its eccentricity, and i is its inclination. These momenta are conjugate to the mean anomaly l, argument of perihelion ω, and the longitude of the ascending node Ω. J_z is the component of orbital angular momentum normal to the plane defined by the binary.

As the system is now azimuthally symmetric J_z is a conserved quantity; the problem is reduced from three degrees of freedom to two. If the test particle orbit is initially circular and inclined near 90°, J_z will be a small quantity. If the torque from the ring of mass reduces the inclination of the test particle orbit, the eccentricity of the test particle orbit must increase to maintain a constant value of J_z. The resulting large eccentricities lead to close encounters between the test particle and the primary star, at which point the test particle orbit is no longer integrated.

If we also average over the mean anomaly of the test particle orbit, replacing the test particle by an eccentric ring, L (and the semimajor axis) are conserved quantities. The problem is then reduced to one degree of freedom. The trajectories of the system are then simply contours of the phase portrait of J and ω. We have confirmed that the mechanism described above is in fact responsible for the gap seen in Fig. 1 by comparing the results of the full, unaveraged numerical integrations to the contours of the averaged system. They coincide closely despite the assumptions required to simplify the analysis.

The stable regions, as seen when projected onto the plane of the sky, are presented in Figs. 7 and 8. The density of plotted points is proportional to the projected density of planets in the α Cen system if the phase space corresponding to the black cells in Figs. 1, 2, and 5 is uniformly populated with circular orbits. It should be noted that, in Fig. 8, the presence of any dots near or within the binary is, of course, a projection effect, and reflects the real possibility of a planet in an exterior orbit passing across a line of sight near the stars.

The habitable zone for planets, as defined by Hart (1979), lies about 1.2–1.3 AU (1″) from α Cen A. A similar zone may exist 0.73–0.74 AU (0.6″) from α Cen B. More recent calculations are more optimistic, allowing the width of habitable zone to be a few times Hart's value (Kasting *et al.* 1993). These zones are based on illumination of the planet by a single star. The presence of a second source of illumination, together with the time variability associated with an eccentric orbit are likely to affect the positions and widths of the habitable zones (Hale 1996). Nevertheless, it appears likely that habitable planets could exist within the stable zone as defined here.

Our results are in qualitative agreement with Harrington's (1968, 1972) studies of hierarchical triple star systems. He found that the inner binary tended to be unstable when its orbital plane was perpendicular to the orbital plane of the most distant member.

Benest (1988) also investigated the stability of planets in the α Cen system. He only explored the case of zero inclination but did allow for non-zero planetary eccentricities. Though using simulations of shorter duration (100–1000 binary periods), Benest found that circular orbits are stable over a range of initial distances roughly similar to that found here. For example, he found stable initially circular prograde orbits around α Cen B out to roughly 3.7 AU ($0.16a_b$), somewhat larger than our result of 3 AU. Benest also found the stable region to be slightly larger around α Cen A than B, in agreement with our results. However, though Benest found retrograde orbits to be stable out to somewhat larger distances than prograde ones in general, he found the opposite to be the case for initially circular orbits. This is in contradiction to our results, but may reflect the different integration length and stability criterion used, for Benest does not consider a planet unstable unless it suffered collision or escape. It also seems possible, given the non-zero Lyapunov times observed for some apparently stable particles, that the stable region around each of the stars would shrink as integration time scales increase.

The fact that planets would seem to be more stable when in the plane of the binary's orbit may increase the likelihood of planets existing in the α Cen system. If one assumes that the planets form and then remain roughly in the primary's equatorial plane, as they have in our Solar System, the possible coincidence of α Cen's equatorial and orbital planes (Doyle *et al.* 1984; Hale 1994) indicates that, should planetary formation have proceeded in a manner similar to that in which it did here, it is plausible that planets might remain in the system to this day.

6. CONCLUSIONS

Our studies reveal that a single planet on a circular orbit is unstable over much of the region around the central α Cen binary. However, there are zones in which such a planet could be stable over million year time scales. These zones are located both far from ($a \gtrsim 70$ AU) the binary and near to ($a \lesssim 3$ AU) the primary and secondary. Stability is a strong function of the inclination for orbits in the inner regions, less so for orbits exterior to both stars. The inner stable region encompasses Hart's (1979) habitable zone; however, a planet orbiting in the more distant stable region would be inhospitable to life as we know it.

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