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THE ORBITAL EVOLUTION OF NEAR-EARTH ASTEROID 3753

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ABSTRACT

Asteroid 3753 (1986 TO) is in a 1:1 mean motion resonance with Earth, on a complex horseshoe-type orbit. Numerical experiments are performed to determine its medium-term stability and the means by which it may have entered its current orbit. Though 3753 moves primarily under the influence of the Sun and Earth, the giant planets (and Jupiter especially) play an important role by influencing, through torque-induced precession, the position of the asteroid's nodes. Variations in the nodal distance strongly affect the interaction of 3753 with Earth and may change or destroy the horseshoe-like behavior currently seen. This precession of the nodes provides a mechanism for placing minor planets into, or removing them from, a variety of horseshoe-type orbits. The chaotic nature of this asteroid's orbit makes predictions difficult on timescales longer than its Lyapunov time (∼150 yr); therefore, ensembles of particles on orbits near that of 3753 are considered. The asteroid has a high probability of passing close to Venus and/or Mars on 10^4 yr timescales, pointing to a dynamical age much shorter than that of the solar system.

Key words: celestial mechanics, stellar dynamics — minor planets, asteroids

1. INTRODUCTION

Asteroid 3753 (1986 TO) was recently revealed to be in 1:1 mean motion resonance with Earth (Wiegert, Innanen, & Mikkola 1997). This asteroid is unusual in many respects: (1) its orbit, when viewed in a frame corotating with Earth, is of a complex, "horseshoe" type; (2) it is the only known object on a horseshoe orbit outside the system of Saturn's moons; and (3) it is the only known minor planet currently in 1:1 resonance with Earth. The results of a study to determine the mechanism by which 3753 came to be in this orbit, and its possible lifetime, are presented here.

Asteroid 3753 was discovered by D. Waldron, working with R. McNaught, M. Hawkins, & M. Hartley, at Siding Spring, Australia, on 1986 October 10 (Minor Planet Circular 11312 [1986]; Pilcher 1989). The original computation of this asteroid's orbital elements was performed by C. Bardwell, whose identification of this object with the earlier sighted 1983 UH by R. West & G. DeSanctis (Minor Planet Circulars 8526–8527 [1983]) allowed a solid determination of this object's heliocentric two-body elements to be made (Waldron et al. 1986; McNaught & Bardwell 1986). However, 3753 is more precisely described as part of a three-body "horseshoe" interaction involving both the Sun and Earth.

Horseshoe orbits are so named because of their shape in a reference frame that corotates with their accompanying planet. Though they are long-known features of the three-body problem (Brown 1911; Darwin 1912), they received little attention until the latter half of this century, usually as part of a highly idealized three-body problem in which the bodies orbiting the Sun move on coplanar, nearly circular orbits (see, e.g., Thuring 1959; Rabe 1961; Giacaglia 1970; Weissman & Wetherill 1974; Garfinkel 1977; Dermott & Murray 1981a, 1981b; Mikkola & Innanen 1992). However, there have been investigations, usually of a numerical rather than an analytic nature, of cases in which the third bodies had significant heliocentric eccentricities (and more rarely, inclinations), sometimes as part of studies of tadpole orbits (e.g., Schanzle 1967; Deprit & Henrard 1969; Hollabaugh & Everhart 1973; Everhart 1973; Taylor 1981; Zhang & Innanen 1988; Mikkola & Innanen 1990).

Horseshoe orbits are related to the tadpole or Trojan orbits, in which the test particle oscillates about the L4 or L5 Lagrangian point. Both horseshoe and tadpole orbits are in 1:1 resonance with their accompanying body, but horseshoe orbits, when viewed in the rotating frame, encompass the L3 point as well as the L4 and L5 points. Bodies in 1:1 resonance with a planet are not uncommon: Jupiter has many (the Minor Planet Center reported 416 in 1998 February), and Mars one (Kinoshita et al. 1990); comet P/Slaughter-Burnham is librating about a 1:1 resonance with Jupiter (Marsden 1970); and there is a ring of interplanetary dust in resonant lock with our globe (Dermott et al. 1994). However, none of these bodies display horseshoe behavior.

The possibility that known minor planets might evolve into or from horseshoe orbits has been previously recognized (Milani et al. 1989; Michel, Froeschlé, & Farinella 1996), but the only other system currently displaying horseshoe behavior is that of the Saturnian moons Janus and Epimetheus (Synnott et al. 1981; Dermott & Murray 1981b). However, horseshoe orbits have been postulated to be involved in the transfer of satellite debris from Jupiter's satellite system (Agafonova & Drobyshevski 1985) and in some aspects of the behavior of planetary rings (Brahic 1982; Seidelmann, Harrington, & Szebehely 1984; Lissauer & Peale 1986).

Asteroid 3753 is currently a nominal Aten asteroid, as it crosses Earth's orbit and its semimajor axis is less than 1 AU. According to the classification of Milani et al. (1989), 3753 is of the Toro class, meaning that it is an Earth crosser in mean motion resonance with our planet. Asteroid 3753

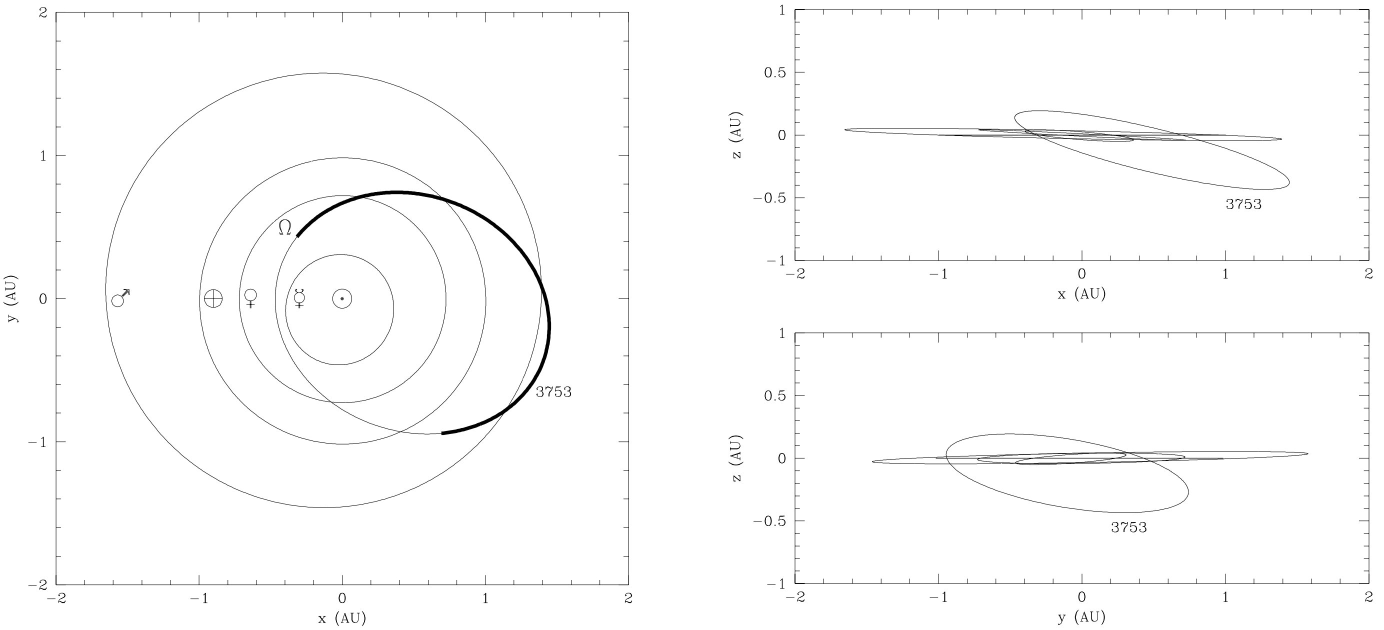

FIG. 1.—Orbits of the inner planets of our solar system and asteroid 3753. The ecliptic defines the *x*-*y* plane, with *x* along the vernal equinox. The heavy line indicates the portion of the asteroid's orbit below the ecliptic, and Ω its ascending node.

also crosses the orbits of Venus and Mars. Little is known of the physical properties of 3753. It has an absolute visual magnitude of 15.1 (Bowell 1997),[1] but its albedo *p* is unknown. Its diameter *D* can be estimated by considering typical asteroidal albedos. If 3753 were an S-type ($p \sim 0.19$) or C-type ($p \sim 0.05$), then *D* would be 3 or 6 km, respectively (Bowell & Lumme 1979; Rabinowitz 1993). A diameter in this range is unexceptional for asteroids in general, but is larger than is typical of near-Earth objects.

Though many advances have been made in the analytic study of the three-body problem (e.g., Szebehely 1967), the situation of asteroid 3753 is particularly difficult. The application of traditional perturbation methods is complicated by the fact that neither the eccentricity nor the inclination of asteroid 3753 is small, and in fact, the high value of the inclination is critical to its behavior. As well, it will be shown that other planets, in particular Venus, Mars, and the Jovian planets, all play key roles in this asteroid's evolution, and thus a three-body description of 3753's motion is incomplete at best. For these reasons, a numerical rather than an analytic approach seems more straightforward and appropriate at this time.

The initial ecliptic heliocentric orbital elements used are from Bowell (1997) and are listed in Table 1. A plot of the orbit of 3753, as seen in the usual nonrotating frame, is presented from different viewing angles in Figure 1.

The numerical algorithm used here is that due to Wisdom & Holman (1991). Our model includes all the planets with the exception of Pluto. No relativistic corrections are applied. The Earth-Moon barycenter has been used rather than simulating these bodies separately. This approximation is valid outside close encounters, and indeed it will be shown that 3753 does not approach Earth to within less than about 40 times the Earth-Moon distance (§ 2). The code does not efficiently handle close encounters, and thus our simulations are terminated should 3753 pass within a planet's sphere of influence R_I:

$$R_I = [M_p/(1\ M_\odot)]^{2/5} r_p\ , \qquad (1)$$

where M_p and r_p are the mass and semimajor axis of the planet, respectively. The spheres of influence of the planets Mercury through Mars have radii of 1.1×10^5, 6.2×10^5, 9.3×10^5, and 5.8×10^5 km, corresponding to approximately 45, 100, 145, and 170 planetary radii, respectively.

The Lyapunov times of 3753 and other planet-crossing bodies are of order 100 yr (Tancredi 1995; Whipple 1995; Wiegert et al. 1997), and thus the simulations over longer timescales are not strictly valid. The phase-space trajectory of a body with the nominal orbital elements given in Table 1 diverges exponentially from that of the real asteroid 3753, and thus one cannot hope to predict the true evolution of this body. However, this phase-space divergence need not necessarily lead to wandering of a body from its orbit, but perhaps simply to large uncertainties in the position of the body within its orbit (see, e.g., Laskar 1989). Thus one can hope to extract information on timescales longer than the Lyapunov time. The effect of chaos is investigated in § 4, where multiple particles, or "clones," each with initial position and velocity differing slightly from the nominal initial conditions of 3753 and from each other, are considered.

TABLE 1

HELIOCENTRIC ORBITAL ELEMENTS OF NEAR-EARTH ASTEROID 3753 USED IN THESE SIMULATIONS

Parameter	Value
Epoch (JD)	2,450,500.5
Equinox	J2000.0
Semimajor axis, *a* (AU)	0.99778030
Eccentricity, *e*	0.51478431
Inclination, *i* (deg)	19.812285
Longitude of the ascending node, Ω (deg)	126.373212
Argument of perihelion, ω (deg)	43.640637
Mean anomaly, *M* (deg)	40.048932

NOTE.—From Bowell 1997.

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[1] See also ftp://ftp.lowell.edu/pub/elgb/astorb.html.

The time step Δt used in the simulations is 2×10^{-4} yr ($\sim$2 hr) unless indicated otherwise, though the gross features of the evolution are not significantly affected by the use of step sizes as large as 10^{-2} yr ($\sim$3 days). The small time step makes the reliable detection of close encounters simple, though it provides no significant increase in accuracy or relief from the effects of chaos. For these relatively short simulations, this choice's ease of implementation outweighs any efficiency gained through more elaborate encounter detection schemes.

To determine whether Δt is small enough to allow reliable determination of close asteroid-planet approaches, let ζ be the ratio of R_I to the distance traveled by the asteroid over one time step, given approximately by

$$\zeta \approx \frac{R_I}{v\,\Delta t} \approx \frac{[M_p/(1\,M_\odot)]^{2/5} r_p}{[GM_\odot(2/r_p - 1/a)]^{1/2}\,\Delta t} = \frac{M_p^{2/5}\, r_p}{2\pi(2/r_p - 1/a)^{1/2}\,\Delta t}\,, \tag{2}$$

where the units are solar masses, astronomical units, and years. For the planets Venus, Earth, and Mars, ζ equals 2.5, 4.9, and 5.5, respectively.[2] The corresponding probability p that a particle will "jump through" the sphere of influence during one time step, based on simple geometric considerations, is approximately

$$p \le 1 - \left(1 - \frac{1}{4\zeta^2}\right)^{3/2}\,, \quad \zeta \gg 1\,, \tag{3}$$

which has values 0.06, 0.02, and 0.01 for Venus, Earth, and Mars when $\Delta t = 2 \times 10^{-4}$ yr. Thus one concludes that close encounters between the asteroid and a terrestrial planet are accurately detected.

General statistical studies of the survival of near-Earth asteroids indicate typical lifetimes of 10–100 Myr, with roughly comparable probabilities of ultimately colliding with Earth, colliding with Venus, or being ejected from the solar system (Greenberg & Nolan 1993). Thus, it seems a priori unlikely that 3753 is a member of a primordial population of objects in 1:1 resonance with our planet. Nevertheless, the relatively unknown character of such highly eccentric and inclined orbits makes such a possibility worth investigating, especially considering its implications for Earth impact rates and near-Earth asteroid research in general.

The current nominal orbit of asteroid 3753 is examined in more detail in § 2; the stability of its 1:1 resonance with Earth is discussed in § 3, the effects of chaos and the probabilities of evolutionary paths are investigated in § 4, other complex 1:1 resonant orbits that exist near that of 3753 are described in § 5, and conclusions are presented in § 6.

2. SHORT-TERM BEHAVIOR OF 3753

The heliocentric orbit of asteroid 3753 is shown in a nonrotating frame in Figure 1, along with those of the inner planets of the solar system. From this figure it is clear that, though the asteroid's current semimajor axis ($a \approx 0.9978$ AU) is very close to that of Earth, it seems inappropriate to call 3753 a "co-orbital" asteroid, as its orbit is very unlike that of our planet in most other respects. Asteroid 3753 has a relatively high eccentricity (0.515) and inclination (19°.8) and crosses the orbits of Venus, Earth, and Mars. There is little in Figure 1 to indicate that 3753 might be in a special relationship with any of the inner planets; however, its connection with Earth becomes apparent when the time evolution of the asteroid's semimajor axis is examined. The semimajor axis of the asteroid evolves smoothly from near 0.998 AU to near 1.002 AU at roughly 385 yr intervals (see Fig. 2), these values corresponding to orbital periods slightly below and above that of Earth, respectively. This oscillatory behavior of a about the semimajor axis of Earth is diagnostic of a 1:1 mean motion resonance.

The trajectory of asteroid 3753 over a single year has a kidney shape in a frame that corotates with Earth, owing to the asteroid's eccentricity. The path does not close on itself, because the asteroid's period ($\tau \approx 0.99667$ yr) is currently slightly less than that of Earth. Thus, the asteroid advances relative to Earth by roughly 1° (net) per year, its trajectory tracing a spiral along Earth's orbit. This behavior is unexceptional in itself and is qualitatively similar for any near-Earth asteroid that does not interact strongly with our planet. Asteroid 3753 differs from other known near-Earth asteroids in that interactions with Earth result in regular changes in the asteroid's semimajor axis and period, and that these changes result in a smooth, oscillatory motion of the asteroid relative to Earth, over a timescale of centuries.

Each transition of the semimajor axis across $a = 1$ AU produces a reversal of the net motion of the asteroid relative to Earth. The particle spirals along Earth's orbit, each time effectively repelled by near approaches to our planet. The position of 3753, plotted at intervals over a single half-cycle and projected onto the ecliptic plane, is shown in Figure 3. The inclination of its orbit allows an apparent overlap with Earth's position from this point of view, at which times the asteroid passes far "below" us, i.e., it is in the far southern skies.

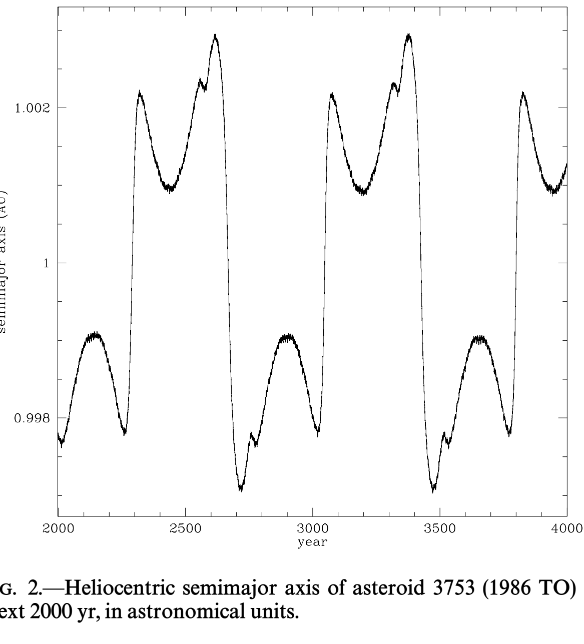

FIG. 2.—Heliocentric semimajor axis of asteroid 3753 (1986 TO) over the next 2000 yr, in astronomical units.

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[2] Mercury's small sphere of influence makes it impractical to choose a step size small enough to ensure close-encounter detection, though the perihelion of 3753 will drop slightly below this planet's aphelion distance (0.48 AU) in the future (see Fig. 7 below). Thus, our simulations effectively assume that there are no close encounters with Mercury. This is an ad hoc assumption, to be investigated in the future. Nevertheless, it appears justified given the small size of Mercury's sphere of influence, and the fact that 3753's node does not cross this planet's orbit over the time frame presented here (Fig. 11).

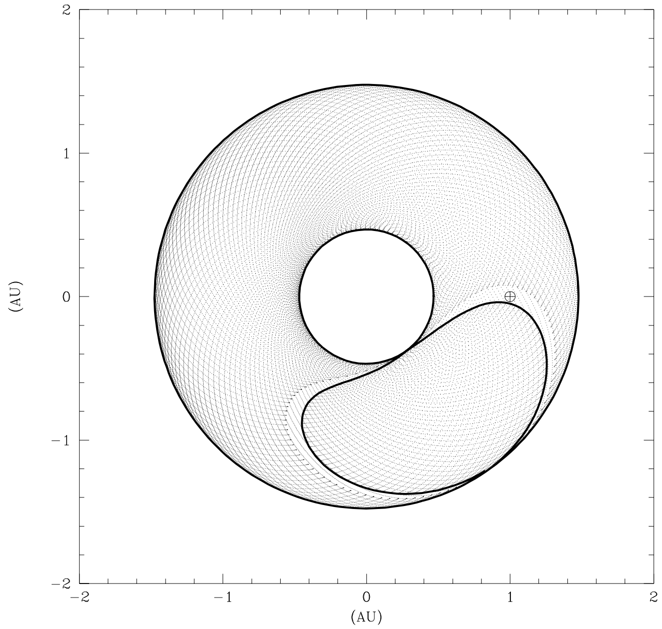

FIG. 3.—Projection of the orbit of asteroid 3753 onto the ecliptic plane, in a frame corotating with Earth, whose approximate position is indicated by the circled plus sign. The heavy outer line indicates the outer edge of the asteroid's horseshoe orbit, while the lighter points indicate the position of the asteroid at 0.01 yr intervals over one half-cycle (approximately 385 yr). The less shaded areas near the ends of the horseshoe are a result of the details of the reversal process. The axes are in astronomical units, and the Sun is located at the origin.

The kidney-shaped motion can be removed by plotting the asteroid's mean position, averaged over 1 year (Fig. 4). The resulting plot shows the distinctive horseshoe shape, but with the opening offset from Earth's position, again because of the overlap allowed by the high inclination of 3753.

The (unaveraged) difference $\Delta\lambda = \lambda_{3753} - \lambda_{\text{Earth}}$ between the ecliptic longitude of asteroid 3753 and that of Earth is shown in Figure 5 at intervals of 0.1 yr. The times at which the average rate of change of $\Delta\lambda$ changes sign (hereafter "reversals") correspond to the ends of the horseshoe. A full horseshoe cycle requires about 770 yr to complete. The previous two reversals occurred roughly in 1900 and 1515 C.E.; the next two will occur in 2285 and 2670 C.E. At one end of the horseshoe, $\Delta\lambda$ approaches zero from negative values but does not reach it. However, during the subsequent reversal, the asteroid passes through the ecliptic longitude of Earth and proceeds onward before reversing direction again. This passage through zero longitude difference creates the overlap with Earth's position seen in Figure 3. Thus asteroid 3753 librates through an angle of almost 420°, and this motion is centered 150° ahead of Earth rather than on the L3 point.

The asteroid's inclination is high enough to allow it to pass through Earth's ecliptic longitude without necessarily suffering a close encounter that might strongly perturb its orbit. However, close encounters do seem to drive the horseshoe behavior of this asteroid. The reversals occur near the times of closest approach, implying that the strengthening gravitational interaction between 3753 and Earth is at work. It should be noted that the closest approaches do not occur when the asteroid and planet have similar longitudes. Rather, the closest approaches occur when the asteroid is at its descending node and Earth is nearby. The ascending node $r_+ = 0.53$ AU is far from Earth's orbit and the asteroid does not interact strongly with our planet there, while the descending node $r_- = 1.17$ AU is relatively close, and the closest approaches occur near this point. During these approaches, which are no closer than 0.1 AU (or about 40 Earth-Moon distances),

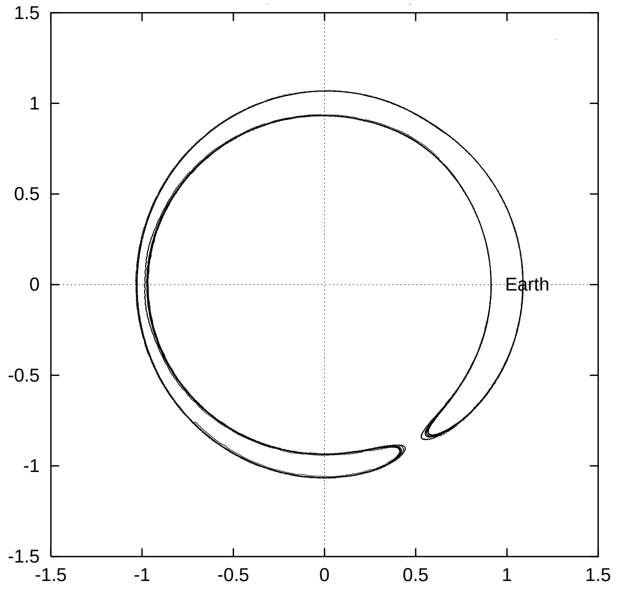

FIG. 4.—Mean motion (averaged over 1 yr) of asteroid 3753 with respect to Earth in a corotating frame. The deviation of the asteroid's radial distance from that of Earth has been exaggerated by a factor of 30.

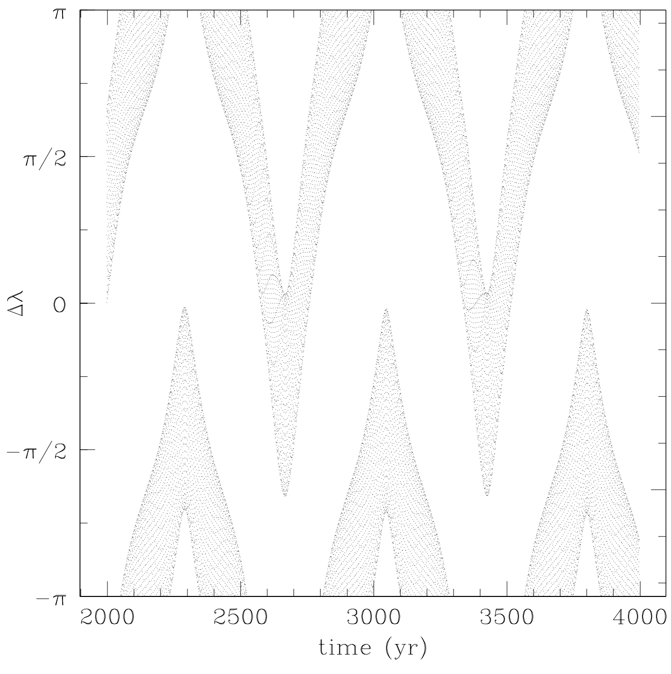

FIG. 5.—Ecliptic longitude of asteroid 3753 relative to that of Earth for the next 2000 yr, plotted at approximately 0.1 yr intervals. The next two closest approaches to Earth occur near 2285 and 2670 C.E., during which the direction of the asteroid's progression in longitude difference reverses. The second such approach occurs during the overlapping leg of the horseshoe; near this time, the asteroid frequently passes through the ecliptic longitude of Earth.

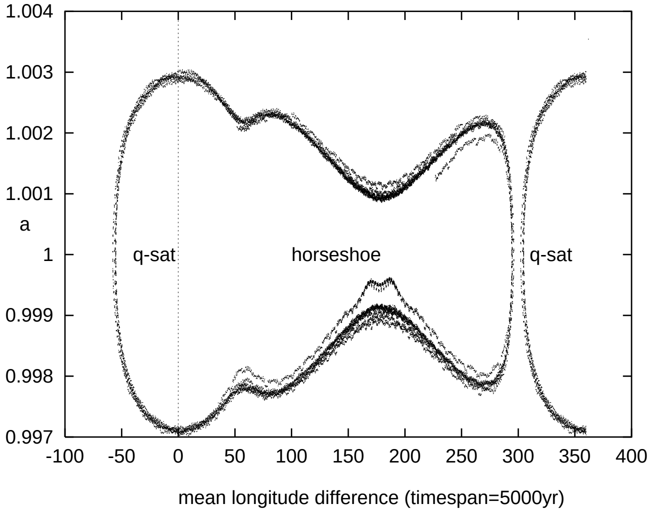

FIG. 6.—Semimajor axis vs. the mean longitude difference (averaged over 1 yr) between asteroid 3753 and Earth.

energy and angular momentum are transferred to or from the asteroid, resulting in orbital changes intrinsically similar to those of coplanar horseshoe orbits (Dermott & Murray 1981a). It will be shown in § 3 that, not surprisingly, the positions of the nodes are of great importance in understanding the behavior of this asteroid. In fact, the current behavior of 3753 may be partially described as a libration of the descending node about the L3 point with an amplitude of over 175° (Fig. 8 below).

The semimajor axis versus the orbit-averaged longitude difference is plotted in Figure 6. Typical horseshoe behavior is seen when the mean longitude difference is between 60° and 300°. However, there is a curious-looking loop around 0° ± 60°, where the asteroid is in what has been called a “quasi-satellite” orbit, a term describing an object in 1:1 mean motion resonance and which librates about $\Delta\lambda \approx 0°$ (Mikkola & Innanen 1997). However, the apparent transfer between quasi-satellite and horseshoe orbits is previously unknown both in theory and in nature, and is a result of the libration of the nodes.

The horseshoe behavior of 3753 is quite complex and naturally suggests that other complex orbits may exist at high heliocentric inclinations and/or eccentricities. A few such orbits near that of 3753 in phase space will be described in § 5.

3. ORBITAL EVOLUTION

The current horseshoe behavior of 3753 is primarily a three-body phenomenon and can be approximated for some purposes by examining the asteroid's trajectory in the presence of only Earth and the Sun. However, the long-term behavior and stability of the orbit are strongly affected by the other members of the solar system, including the terrestrial and Jovian planets.

A plot of the orbital elements of 3753 over a roughly 20,000 yr interval centered on the present is shown in Figure 7. The Lyapunov time of 3753 has been found to be roughly 150 yr (Wiegert et al. 1997). Since the asteroid's orbit is chaotic, its true phase-space trajectory will diverge exponentially from that computed in our simulations. However, integrations of over 100 particles with initial conditions near those used for 3753 (given in Table 1) show very similar secular behavior of the orbital elements over the whole simulation length. The exception is the semimajor axis a, which typically shows a different pattern of oscillation at times $t \gtrsim 5000$ yr and $t \lesssim -3000$ yr (though the absolute value of a usually remains close to its current value). We also note that the simulation displayed in Figure 7 ends, both in the forward and reverse directions, with a close encounter between the asteroid and a planet, and that

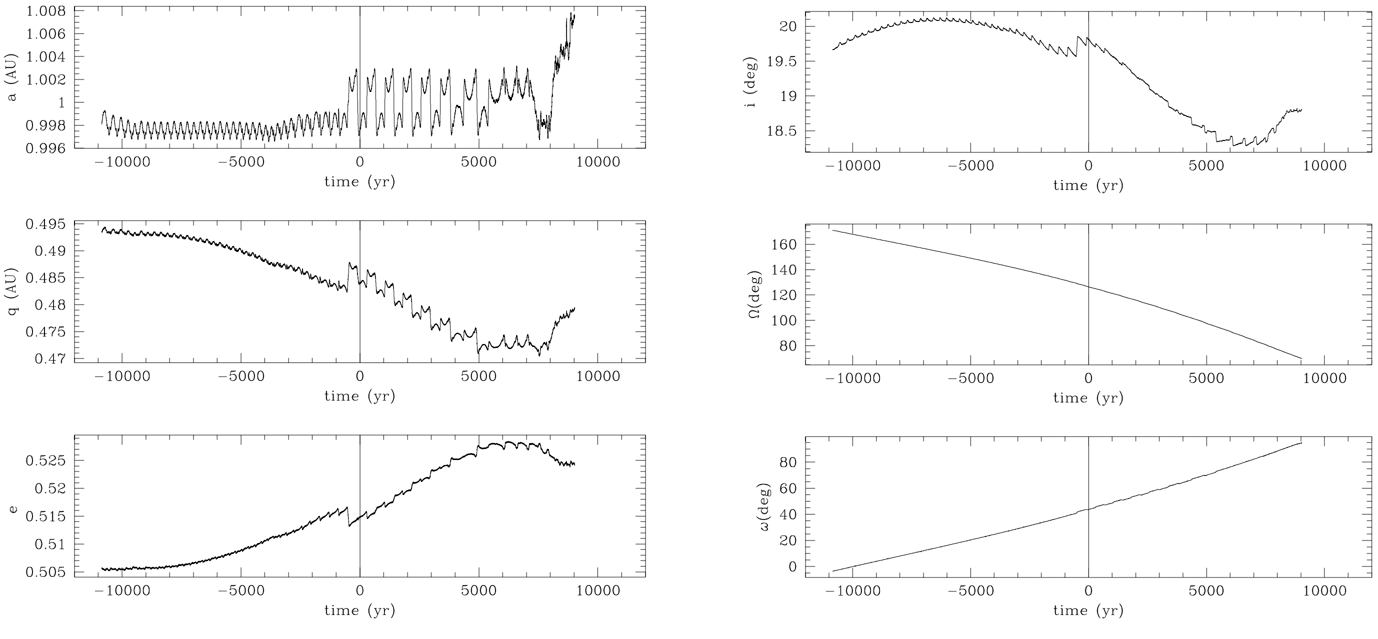

FIG. 7*a*

FIG. 7*b*

FIG. 7.—Osculating heliocentric orbital elements (*a*) a, q, and e and (*b*) i, Ω, and ω of asteroid 3753 over roughly 20,000 yr centered on the present. The simulations were terminated in the future after roughly 9000 yr at a close encounter with Venus, and in the past after roughly 11,000 yr at a close encounter with Mars.

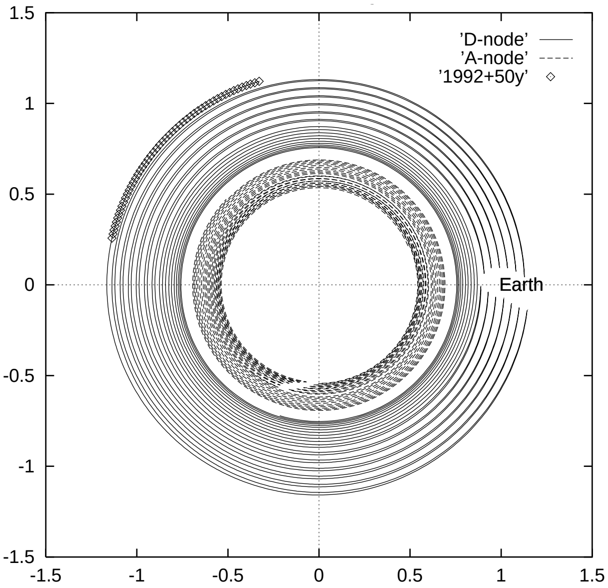

FIG. 8.—Positions of the ascending and descending nodes, plotted in a corotating coordinate system over a total time span of 7500 yr. The diamonds indicate the location of the descending node over a 50 yr span beginning at 1992.3.

the occurrence of encounters is very sensitive to the uncertainties in the initial conditions. Thus Figure 7 does not represent the precise evolution of asteroid 3753, but provides useful insights into its probable behavior. A statistical investigation of asteroid-planet encounters is presented in § 4.

The plot of the osculating heliocentric semimajor axis (Fig. 7) suggests that the horseshoe behavior is not stable over 10^4 yr timescales, and an examination of the asteroid's trajectory shows this to be the case. The asteroid began its current mode of behavior only 500 yr ago and will leave it in another 5000 yr. Though the asteroid's orbit is chaotic, the transition between types of behavior is not entirely unpredictable, but is largely governed by secular changes in the asteroid's angular elements, particularly the argument of perihelion ω.

The elements e, q, i, Ω, and ω are subject to relatively large, long-period perturbations, while variations on the timescale of the horseshoe libration period are comparatively small (Fig. 7). The long-period evolution of the angular orbital elements is due primarily to torques exerted by the other planets in the solar system, especially Jupiter. The effects of the giant planets were determined by performing simulations in which some or all of them were absent.

The net rates of change of e and i are relatively small ($\lesssim 0.1\%$ per 1000 yr), and their evolution has little qualitative effect on the asteroid's trajectory over the timescales considered here. However, the precession of ω ($\dot{\omega} \approx +0\overset{\circ}{.}5$ per century) does play a key role in 3753's evolution. The longitude of the ascending node Ω also varies at a rate close to that of ω, but in the opposite direction: this serves to preserve the longitude of perihelion of 3753's orbit near $\varpi = \Omega + \omega \approx 170°$. The import of this is unclear at this time and may be relatively unimportant to the question of stability, which is known to depend most strongly on the radial distance of a planet-crossing asteroid's nodes (Shoemaker et al. 1979).

The distance between the Sun and the nodes is given by $r_\pm = a(1 - e^2)/(1 \pm e \cos \omega)$, where the plus and minus signs indicate the ascending and descending nodes, respectively. The relative rate of change of the nodal distances $\dot{r}_\pm$ is

$$\frac{\dot{r}_\pm}{r_\pm} = \frac{\dot{a}}{a} - \frac{2e\dot{e}}{1 - e^2} \pm \frac{\dot{e} \cos \omega - e\dot{\omega} \sin \omega}{1 \pm e \cos \omega}\,, \qquad (4)$$

where the upper sign indicates the ascending node. Changes in ω dominate those in a and e for 3753 and largely control the positions of the nodes. As the distances between the nodes and Earth's orbit change, 3753's motion exhibits distinctly different forms.

Currently $\omega \approx 44°$, but it is precessing at a rate of $+0\overset{\circ}{.}5$ per century. The asteroid shows its previously described horseshoe behavior over the range $40° \lesssim \omega \lesssim 70°$, or 1.2 AU $\lesssim r_- \lesssim$ 0.9 AU (the descending node is at 1 AU when $e = \cos \omega$, or $\omega \approx 59°$). Ignoring for the moment the possibility of close encounters with any of the planets, the asteroid would precess through a range of values of ω, exhibiting horseshoe behavior when the values of ω were appropriate and other behavior at other times. At values of $\omega \lesssim 40°$, the asteroid simply circulates (albeit unevenly) through all longitudes; at $\omega \gtrsim 70°$ the trajectory remains a horseshoe, but now opening 120° *behind* Earth in its orbit. The variation in the asteroid's orbit with ω is discussed further in § 5.

During the current horseshoe-like era, the descending node librates (in the rotating frame) in a horseshoe pattern of its own (see Fig. 8). In addition, the radial distance of the descending node, r_-, is decreasing as a result of the asteroid's precession in ω. This precession will bring r_- through Earth's orbit and inward to even smaller values. As the distance between 3753's descending node and Earth's orbit increases, the dynamical link between the two will weaken. Eventually, 3753 and our planet will decouple, and the motion thereafter will become circulation rather than libration. Note that this precession thus "explains" how the transition from one complex horseshoe-type orbit to another occurs and also gives us a good idea how the capture to this kind of orbit happens. An object need not be placed directly into such an orbit (by a close planetary encounter, say) but can evolve in and out of such orbits as a result of their precession-induced motion along an effectively one-dimensional manifold in phase space.

Asteroid 3753 will only evolve through a finite range of ω before its orbit intersects that of a planet, at which point a strong interaction may occur. In the simulations presented in Figure 7, 3753 suffers a close encounter with Venus approximately 9000 yr in the future; in the past, 3753 passed near Mars roughly 11,000 yr ago. These encounters may affect the orbit of this asteroid relatively strongly, though the possibility of a collision at these times seems remote. Unfortunately, the chaotic nature of the orbit of 3753 makes the prediction of close encounters (and their consequences) impossible over such long time spans. A statistical approach to predicting the close-encounter probabilities is therefore adopted in the next section.

4. STATISTICAL RESULTS

Studies of the origin and stability of asteroid 3753 are complicated by the sensitivity of simulations of its orbit to small errors in the initial conditions; thus a statistical approach will be adopted here. Rather than vainly attempting to determine the precise evolution of the asteroid's orbit, its *probable* evolution will be assessed.

The behavior of two ensembles of 64 test particles with orbital elements near those determined for 3753 is investigated. Each set is constructed by applying a small "error" Δ to the particle's position and/or velocity. The following combinations are considered: each particle can have an offset of $+\Delta$ or $-\Delta$ along each of six axes (three of position and three of velocity), for $2^6 = 64$ possibilities.

The "errors" used are arbitrarily chosen to be a fraction ϵ of 3753's initial orbital radius r and velocity v (e.g., $\Delta = \epsilon r$ or ϵv) as determined from the elements in Table 1. The two values of ϵ used are 10^{-5} and 10^{-6}, for a total of 128 particles. The ephemeris uncertainty of 3753 (Bowell 1997) is $0''.29$, corresponding roughly to 10^{-6} AU at a viewing distance of 1 AU.

Consider spheres of radius R_I and $5R_I$ (recall that R_I is the radius of the sphere of influence; eq. [1]) about each of the planets. Passages through these spheres are indicative of strong gravitational interactions, though the exact details thereof may be unclear. The times at which the test particles first pass within these distances of the planets Venus, Earth, and Mars are shown in Figures 9 and 10. No such encounters take place with any other planets. These simulations are run with the time steps 2×10^{-4} and 10^{-3} yr for the smaller and larger spheres, respectively.

All 128 particles passed within $5R_I$ of Mars 2500 ± 500 yr ago and will pass through Venus's $5R_I$ sphere 8000 ± 1000 yr from now (Fig. 9); thus it is possible that 3753 also interacted or will interact with these planets at these times. The occurrence of close encounters is consistent with the positions of the nodes (Fig. 11). In particular, the eccentricity of 3753's orbit is such that both nodes are in the vicinity of Venus's orbit at the same time, doubling the possibility of close encounters during this period.

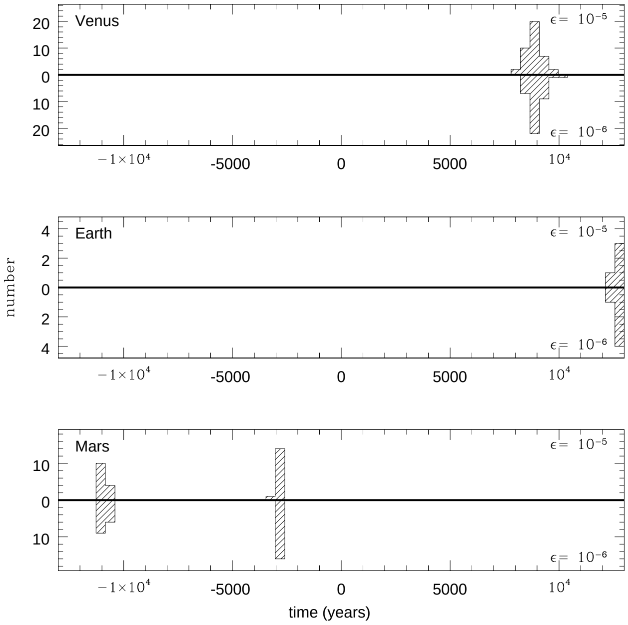

FIG. 10.—Histograms of the times of first passage within the spheres of influence of the planets Venus, Earth, and Mars for the 128 hypothetical particles discussed in § 4 over a 26,000 yr interval centered on the present.

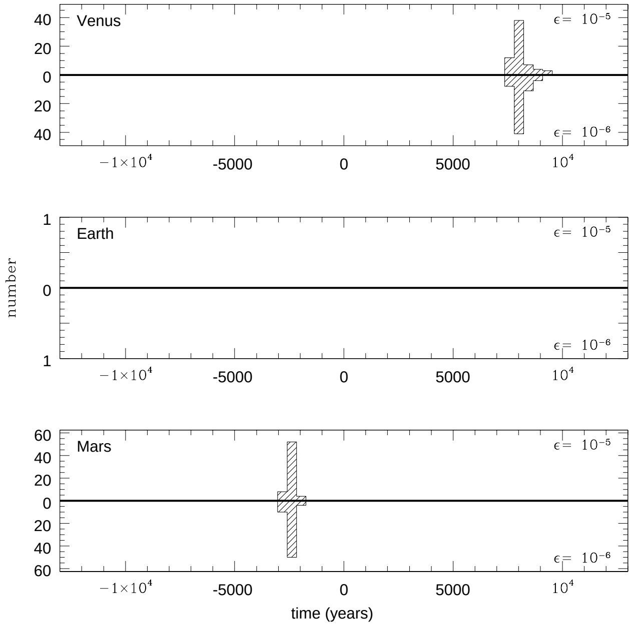

FIG. 9.—Histograms of the times of first passage within 5 times the spheres of influence of the planets Venus, Earth, and Mars for the 128 hypothetical particles discussed in § 4 over a 26,000 yr interval centered on the present.

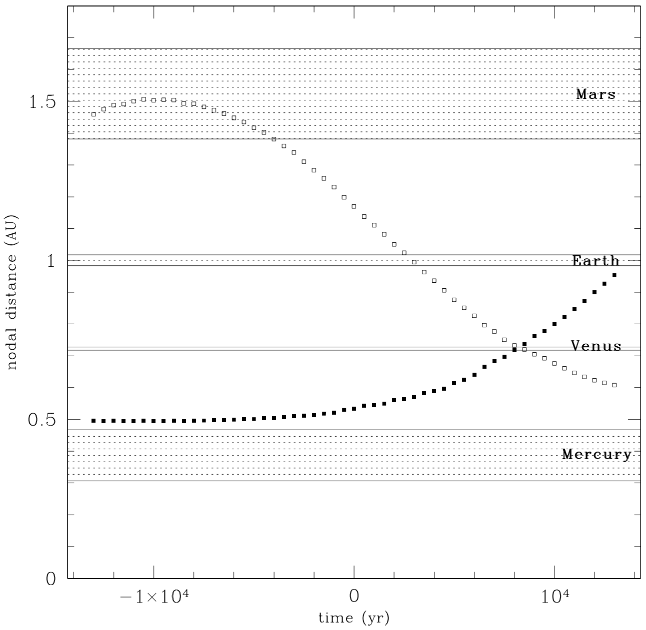

FIG. 11.—Positions of the nodes of asteroid 3753 in the absence of close encounters with any planets. The open and filled squares denote the descending and ascending nodes, respectively. The range of distances spanned by each of the inner planets is shown by the dotted bands.

However, not all particles suffer a passage within the sphere of influence R_I itself on the same timescales. Figure 10 displays the times the clones first pass within R_I of the planets Venus, Earth, and Mars over a 26,000 yr interval centered on the present: 38 of the 128 particles did not pass within R_I of any of the planets when followed 13,000 years into the future; 68 escaped such encounters over simulations covering an equal time span into the past. Of these, 21 particles do not pass within R_I of any planet over the entire 26,000 yr span. Thus, though the ultimate future of 3753 depends strongly on the precise nature of its current orbit, given the high probability of close encounters implied by

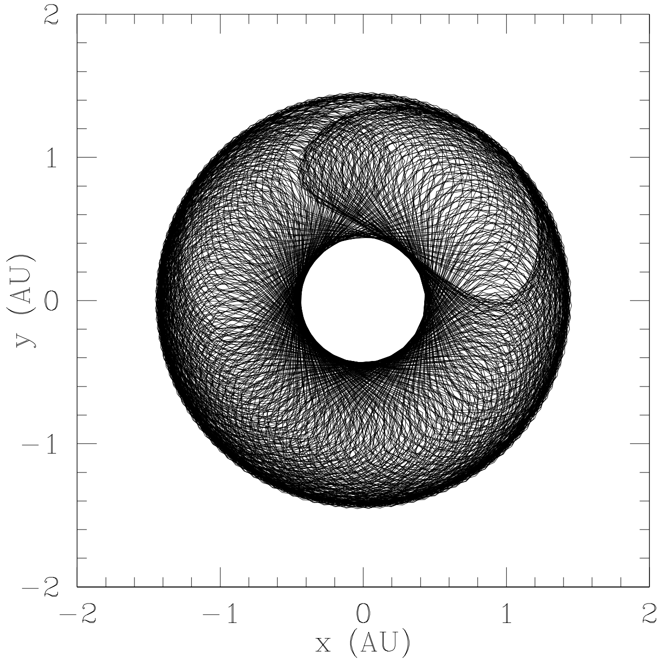

FIG. 12*a*

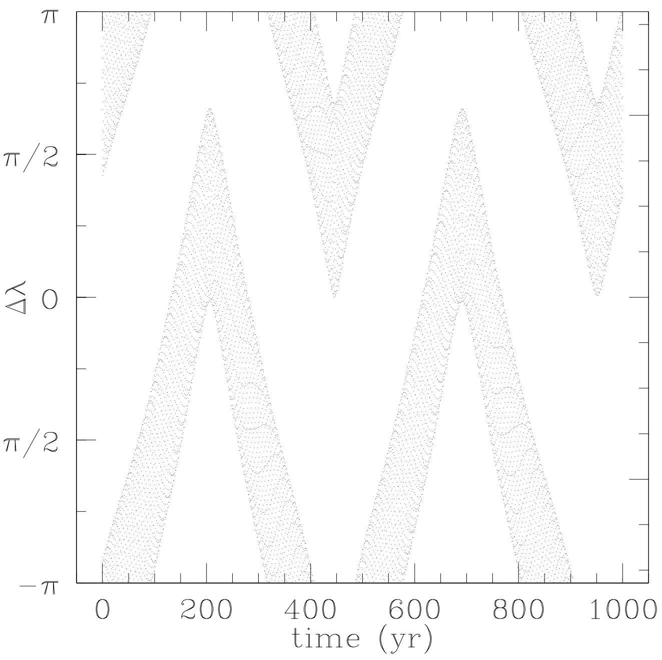

FIG. 12*b*

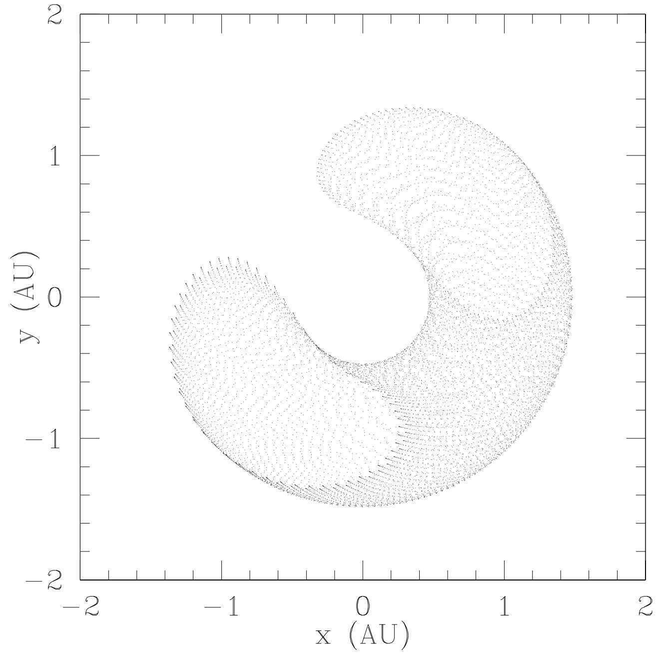

FIG. 12*c*

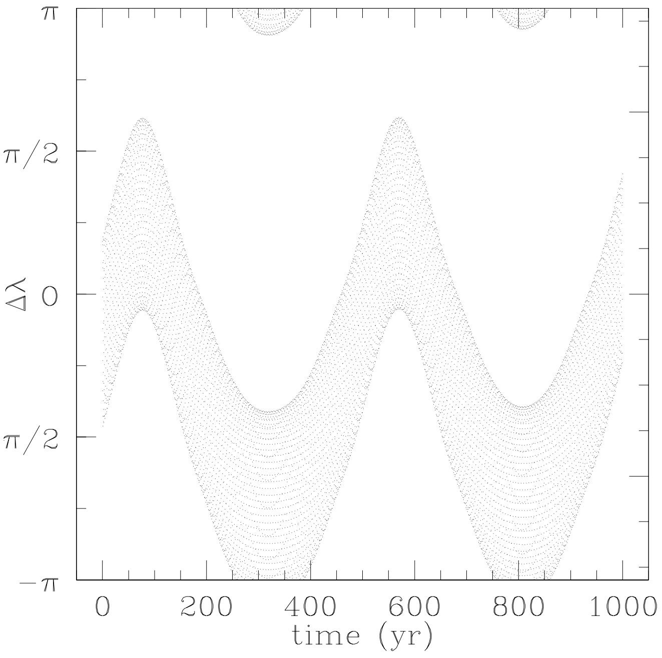

FIG. 12*d*

FIG. 12.—Projection onto the ecliptic plane in the rotating frame [Earth is located at $(x, y) \approx (1, 0)$] and the longitude difference $\Delta\lambda$ between the particle and Earth for orbits with the same orbital elements as 3753 save the argument of perihelion ω, which has the values (*a*, *b*) 120°, (*c*, *d*) 320°, (*e*, *f*) 0°, and (*g*, *h*) 180°. The density of points has been enhanced in (*a*) to clarify its structure: this orbit is a mirror image of that of 3753, reflected about the *x*-axis.

these simulations it seems unlikely that 3753 could have survived in its current configuration for timescales much longer than 10^4 yr. Thus, it seems equally unlikely that 3753 is part of a group of asteroids that have inhabited the 1:1 resonance with our planet since the solar system's formation. However, the possibility that the actual asteroid 3753 is longer lived than the average clone cannot be excluded. In particular, a few of the clones appear to enter the Kozai resonance while remaining in the 1:1 mean motion resonance, a phenomenon that can increase an asteroid's dynamical lifetime (Michel & Thomas 1996).

We also note that the simulations indicate the possibility of a close encounter between 3753 and Earth approximately 12,000 yr in the future. The danger of a collision, though certainly extraordinarily low, should be reevaluated at a future date.

5. ORBITAL FORMS AS A FUNCTION OF ω

The position of the nodes plays an important role in determining the interaction of 3753 with Earth. The role of the nodes is investigated by examining orbits with the same orbital elements as 3753 but with different values of ω.

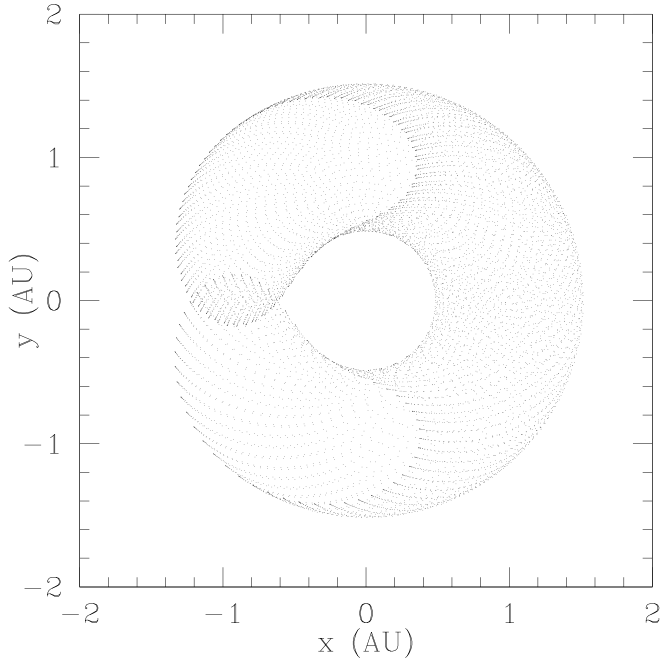

FIG. 12*e*

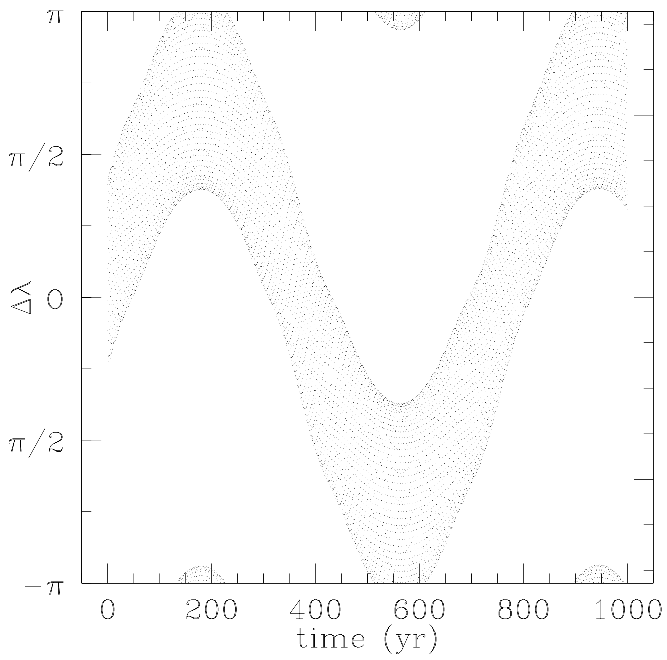

FIG. 12*f*

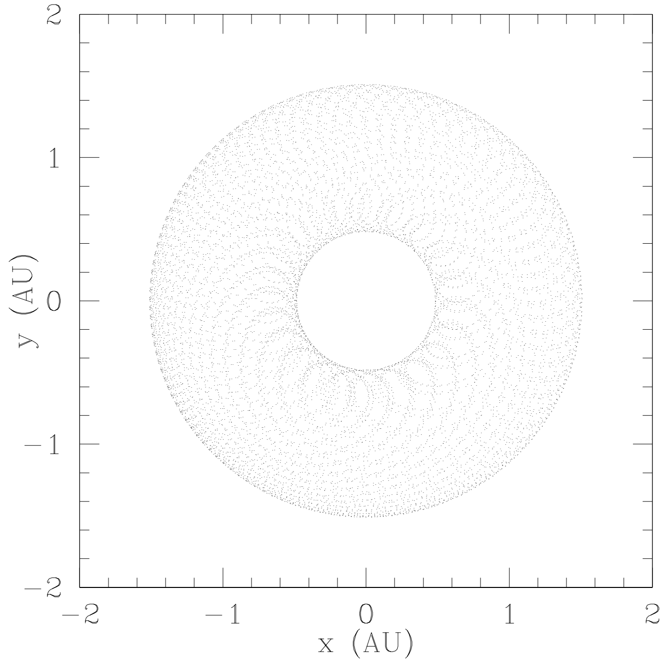

FIG. 12*g*

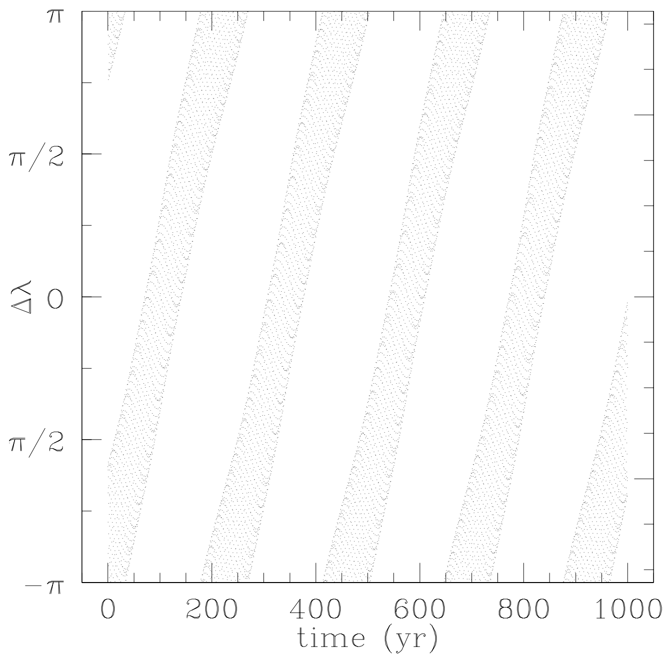

FIG. 12*h*

Values of $0° \leq \omega < 360°$ at 10° intervals have been monitored for periods of a few thousand years, in a three-body (Sun, Earth, particle) system. A variety of qualitatively different forms emerge, which can typically be classified into one of three categories: (1) horseshoe variants, (2) orbits that do not interact strongly with Earth, and (3) complex transitional forms. The horseshoe variants are not uncommon, and constitute roughly half the orbits tested.

Plots of the relative longitude of the particle with respect to Earth, along with projections of the particle's orbit onto the ecliptic, are presented for a sample of forms in Figure 12. The samples include a mirror image of 3753 reflected about the x-axis and horseshoes with openings directed roughly 120° and 180° from Earth [note that in all cases Earth is located at $(x, y) \approx (1, 0)$]. These forms are more difficult to interpret in terms of interactions with Earth, as reversals sometimes occur when the asteroid is quite far from the planet. In this characteristic, these forms are similar to tadpole orbits.

The effect of a varying inclination on 3753's orbit was also examined, again at 10° intervals. At zero inclination, the asteroid quickly encounters Earth. The current horseshoe is exhibited over the range $10° \lesssim i \lesssim 20°$ with a slightly larger amplitude in a (and hence shorter horseshoe period) at the smaller inclination. At $i = 30°$, a horseshoe open at 120° behind Earth is displayed; one open at 180° is exhibited for $40^\circ \lesssim i \lesssim 60^\circ$. The rest of the inclination range shows simple horseshoe-free circulation until large values ($i \gtrsim 170^\circ$), at which various horseshoe behaviors are seen again. The general forms displayed are similar to those seen under the variation of ω. The amplitude of the variations in a seen in these simulations is similar but not identical (i.e., within a factor of 2), and the horseshoe periods are thus also of the same order.

The stability of these orbits is unclear and has only been investigated over a time span of 1000 yr (~1 horseshoe period). Nevertheless, these simulations reveal that a great variety of horseshoe orbits exist over a large fraction of phase space in the three-dimensional three-body problem.

6. CONCLUSIONS

Asteroid 3753 (1986 TO) is a clear example of the complexity of motion that can be exhibited by purely gravitating bodies within the solar system. Though its three-body motion with the Sun and Earth is of primary importance, the planets Venus, Mars (close encounters), and Jupiter (torques) all have key roles to play in the evolution of asteroid 3753. This asteroid also serves as an example of how the behavior of chaotic systems may sometimes be followed (in a statistical sense, at least) over times much longer than its Lyapunov timescale.

The horseshoe behavior currently displayed by 3753 may have started as recently as 500 yr ago and may end 5000 yr from now as a result of the precession of its nodes induced by the giant planets, though the asteroid has likely resided near the 1:1 resonance for a much longer time. The motion of the nodes, and particularly their proximity to Earth and other inner planets, largely governs the longer term behavior and stability of 3753. This nodal precession may result in 3753 becoming decoupled from our planet in the future as the descending node moves away from Earth's orbit. This method of decoupling is distinct from that which occurs when a body's semimajor axis/orbital period is perturbed away from commensurability. Also, we note that precession will transform 3753's current horseshoe orbit into a different but similar one, also in 1:1 mean motion resonance with our planet. Thus, precession of the nodes provides an elegant mechanism of moving particles in and out of horseshoe-type orbits.

On longer timescales, the asteroid seems to have a better than even chance of passing within the spheres of influence of one of the inner planets over a 10^4 yr time span. Such encounters are possible again largely because of the giant planet–induced precession of the nodes, which causes 3753's path to intersect that of different inner planets at various times. The ultimate result of such events is impossible to predict, but they are likely indicative of instability in the asteroid's orbit on a similar timescale. If the asteroid's orbit is relatively short-lived, then it almost certainly entered its current configuration recently as well and is not a member of a primordial population of such asteroids. However, the mechanism by which such an unusual orbit might be produced on such a short timescale is not clearly identified in our simulations, as a result of the strong chaos inherent in the asteroid's behavior.

Complex three-dimensional horseshoes are not uncommon in the phase space near that occupied by 3753, and thus such orbits may not be as rare as they might first appear. Their behavior depends critically on the position of their nodes relative to Earth's orbit. Asteroids may also be transferred from one type of horseshoe behavior to another, through the precession of their arguments of perihelion. This potential for evolving into and out of horseshoe orbits makes it quite possible that other near-Earth asteroids perform similarly; however, at this writing, no other minor planet is known to have a semimajor axis as near to that of Earth as does 3753. Nevertheless, given the apparently large regions of phase space displaying horseshoe behavior, we predict that more objects will be found on such complex trajectories in the future.

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