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# Long-term evolution of the Neptune Trojan 2001 QR322

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## ABSTRACT

We simulated more than a hundred possible orbits of the Neptune Trojan 2001 QR322 for the age of the Solar system. The orbits were generated randomly according to the probability density derived from the covariance matrix of the orbital elements. The test trajectories librate around Neptune's $L_4$ point, with amplitudes varying from 40° to 75° and libration periods varying from 8900 to 9300 yr. The $\nu_{18}$ secular resonance plays an important role. There is a separatrix of the resonance so that the resonant angle switches irregularly between libration and circulation. The orbits are chaotic, with observed Lyapunov times from 1.7 to 20 Myr, approximately. The probability of escape to a non-Trojan orbit in our simulations was low, and only occurred for orbits starting near the low-probability edge of the orbital element distribution (largest values of initial semimajor axis and small eccentricity). This suggests that the Trojan may well be a primordial object.

**Key words:** celestial mechanics – minor planets, asteroids – Solar system: general.

## 1 INTRODUCTION

According to the Minor Planet Center, 1571 Trojan asteroids have been discovered. This inventory includes 1564 Jovian Trojans, six Martian Trojans, and one Neptunian Trojan. The Earth has one temporary co-orbital object, 3753 Cruithne (Wiegert, Innanen & Mikkola 1997) and one horseshoe object, 2002 AA29. So far, no Trojan asteroids have been found for the other planets. Studies by Mikkola & Innanen (1992), de La Barre et al. (1996) and Brasser & Lehto (2002) have shown that Trojans of Mercury, Saturn and Uranus are not expected. Tabachnik & Evans (2000) and Brasser & Lehto (2002) have shown that Venus and Earth Trojans are stable for low inclinations.

We greet the discovery of QR322 with great interest, and although its observed arc is still short, we believe that a preliminary analysis of its motion is justified. Indeed, as we shall demonstrate, its motion is already clear enough to reveal some interesting dynamical features.

## 2 EXPERIMENTS

The orbital elements of QR322 in the heliocentric frame are taken from AstDys[1] and are given in Table 1. Becausee the orbital elements have uncertainties, we computed one hundred orbits generated using the covariance matrix $\mathbf{C}$ from the AstDys pages. The variations $\delta\boldsymbol{q}$ in the element vector $\boldsymbol{q}$ were computed as

$$\delta\boldsymbol{q} = \sum_{k=1}^{6} \xi_k \sqrt{\lambda_k}\boldsymbol{X}_k, \tag{1}$$

where $\lambda_k$ are the eigenvalues of the covariance matrix $\mathbf{C}$ and $\boldsymbol{X}_k$ are the normalized eigenvectors; $\xi_k$ are random numbers with a nearly Gaussian distribution with an rms value equal to unity. (We used the sum of three uniformly distributed random numbers.) In the simulations, we included only the effects of the giant planets. The Sun and the terrestrial planets were placed in their barycentre. The computations were performed using the Wisdom–Holman algorithm (Wisdom & Holman 1991) with a time-step of 400 d for up to 5 Gyr backward in time. (Some less extensive simulations forward in time gave similar results.) To check the sensitivity of the results upon the method of integration, one set (set one) of 50 orbits used the symplectic corrector (Wisdom, Holman & Touma 1996; Mikkola & Palmer 2001), whereas the other set did not. In order to check our results even more thoroughly, we performed another computation of the orbit using a Cowell integrator with the first sum (Huang & Zhou 1993) of order 13 with a step-size of 35 d. The results from all these integrations were, however, qualitatively similar and we mainly illustrate the results from set one only.

To find possible secular resonances, we fed the Laplace–Runge–Lenz vectors and angular momentum vectors of the orbits into a Fourier analyzing routine and inspected the resulting power spectrum.

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[1] http://hamilton.dm.unipi.it/cgi-bin/astdys/astibo

**Table 1.** Orbital elements of 2001 QR322 in the ecliptic, heliocentric frame, taken from AstDys. The epoch for these elements is 2003 July 20 in the J2000.0 system.

| $a$ (au) | $e$ | $i$ (°) | $\Omega$ (°) | $\omega$ (°) | $M$ (°) |
|---|---|---|---|---|---|
| 30.1288 | 0.016885 | 1.326 | 151.707 | 219.671 | 350.353 |

## 3 RESULTS

### 3.1 Coordinate system

When analyzing the simulation results, we computed the orbital elements in the barycentric system, where the *xy*-plane was taken to be the invariable plane of the model Solar system. Thus all the orbital elements discussed were evaluated in this system. Note that the values in this system differ somewhat from those given in the table above.

### 3.2 Overview

Fig. 1 is a plot of the initial distribution of the elements in ($a$, $e$) space. The trajectories that finally escaped are marked with a square, whereas the stable orbits are marked with star. The orbits librate around Neptune's $L_4$ point with amplitudes varying from 40° to 75° and libration periods from 8900 to 9300 yr. The motions are chaotic, with e-folding times of 1.7 to 20 Myr. For most test orbits, the eccentricity varies with a small amplitude and appears to be stable. However, sometimes the long-term behaviour of the inclination is erratic and large fluctuations appear, suggesting the action of a secular resonance.

### 3.3 Evolution of orbital elements.

We found that the proper eccentricity is significantly larger than the forced eccentricity due to Neptune and the other planets. The action of Neptune should force $\varpi \rightarrow \varpi_8 + 60°$ (Message 1966), but because the forced eccentricity is smaller than the proper one, $\varpi$ should circulate, which we observe here. There is a significant number of orbits in the $\nu_{18}$ secular resonance, with the argument $\Omega$–$\Omega_8 \approx 0$. In Fig. 2, we plot $\dot{\Omega}$–$\dot{\Omega}_8$ as a function of initial semimajor axis for the set of orbits where the symplectic corrector was applied. These results were obtained by considering only the first 100 Myr of the computation. The orbits with largest initial semimajor axes and (also) largest values of $\dot{\Omega}$–$\dot{\Omega}_8$ are unstable.

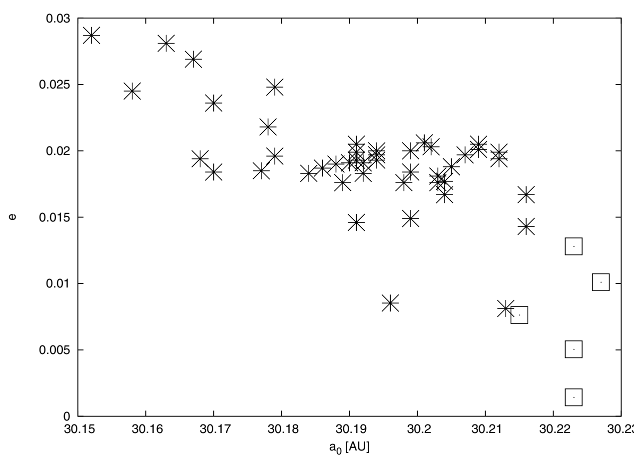


**Figure 1.** Initial distribution of the particles as obtained using the covariance matrix in ($a$, $e$) space in Solar system-barycentric coordinates. Particles that escaped are marked with an asterisk, whereas those that stayed the whole simulation are marked with a square.

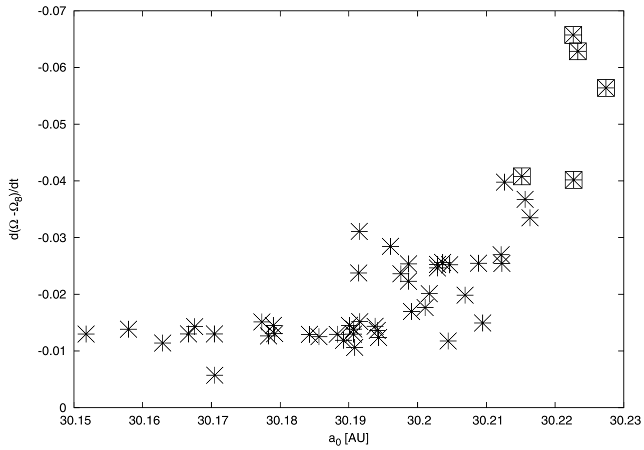


**Figure 2.** The (mean) value of $\dot{\Omega} - \dot{\Omega}_8$ (in seconds of arc/yr) as a function of initial semimajor axis $a_0$. Only those orbits where $\dot{\Omega} - \dot{\Omega}_8$ was significantly larger than average escaped. These are marked with squares. This is the data from experiment set one over the first 100 Myr.

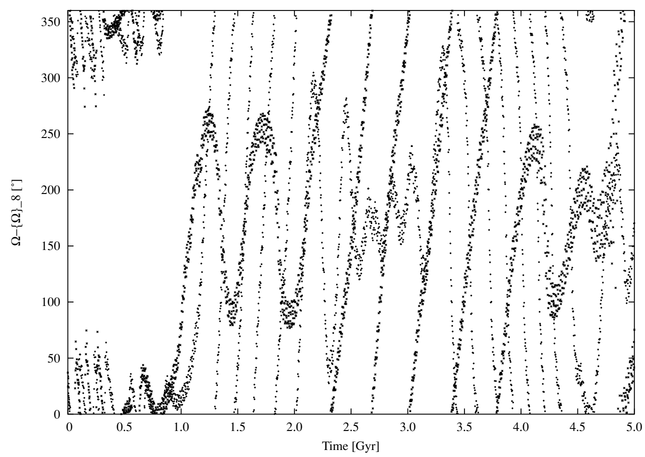


**Figure 3.** Plot of $\Omega$–$\Omega_8$ versus time for two sample orbits; both leave the $\nu_{18}$ resonance. In both cases, the argument $\Omega$–$\Omega_8$ switches between libration and circulation.

Fig. 3 shows the evolution of $\Omega$–$\Omega_8$ in the separatrix for two sample orbits. For one orbit, the argument $\Omega$–$\Omega_8$ begins to circulate at $t \approx 1$ Gyr. At about the same time, the argument $\Omega$–$\Omega_8$ for the other orbit changes its libration to be around 180°. This libration lasts for about 1 Gyr until the argument $\Omega$–$\Omega_8$ starts to circulate too, though it librates again around $t = 3.7$ Gyr. The evolution of the inclinations corresponding to Fig. 3 can be found in Fig. 4. We observed that most orbits that are initially in the separatrix leave it at some point within our 5-Gyr simulations.

### 3.4 Chaos

In nonlinear science, if an orbit falls in the separatrix of a resonance, it may be concluded that the motion is chaotic. From a linear to a nonlinear model, the separatrix becomes a thin chaotic layer (Lichtenberg & Lieberman 1983). The behaviour of falling in a separatrix

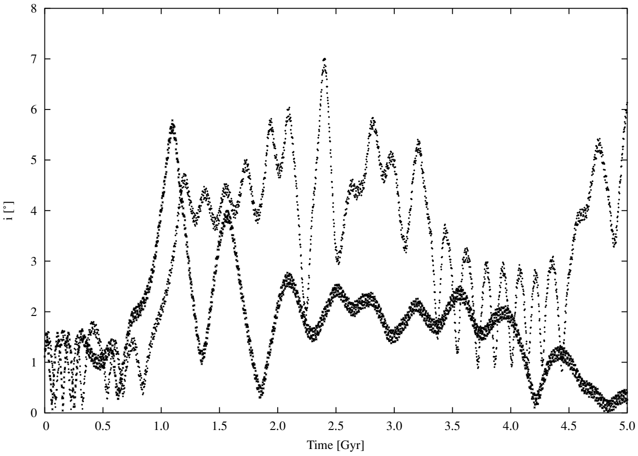


**Figure 4.** Plot of $i$ for the two sample orbits of Fig. 4, both leaving the resonance.

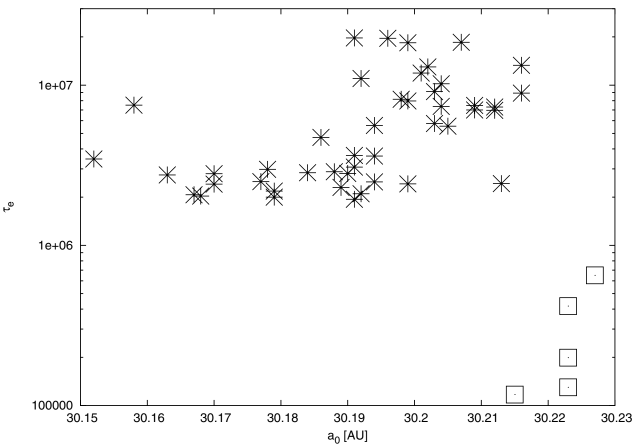


**Figure 5.** The Lyapunov times $\tau_e$ for the orbits of the set one versus initial semimajor axis.

is the switching between circulation and libration. The chaotic separatrix layer is usually thin. This is why the orbits each behave differently and leads us to conclude that the $\nu_{18}$ resonance is the main dynamical factor that governs the motion.

The motions in the test orbits were found to be chaotic, with Lyapunov times $\tau_e$ that vary by an order of magnitude. These were obtained using the tangent map (Mikkola & Innanen 1999) of the Wisdom–Holman integrator. Fig. 5 plots the times $\tau_e$ between neighbouring orbits versus initial semimajor axis. It is clear that those trajectories with $a > 30.195$ au show more regular behaviour than the others. For the other orbits, the argument $\Omega$–$\Omega_8$ only librates around 0, so that they are unaffected by the separatrix, with a period of about 55 Myr. The inclination oscillates with the same period and its typical maximum value is about 1.6°. This is verified in Fig. 6, which shows the maximum obtained inclination versus initial semimajor axis. There is a clear cut-off at $a \approx 30.195$ au for the value of $i_{max}$, showing clearly where the separatrix of the $\nu_{18}$ resonance is located. Furthermore, the number of particles deep inside the $\nu_{18}$ resonance – where it only librates – and in the separatrix is about equal. The few orbits on the edge of the plot which have a very high maximum inclination escaped from the Trojan region, which resulted from their being on the edge of the libration region.

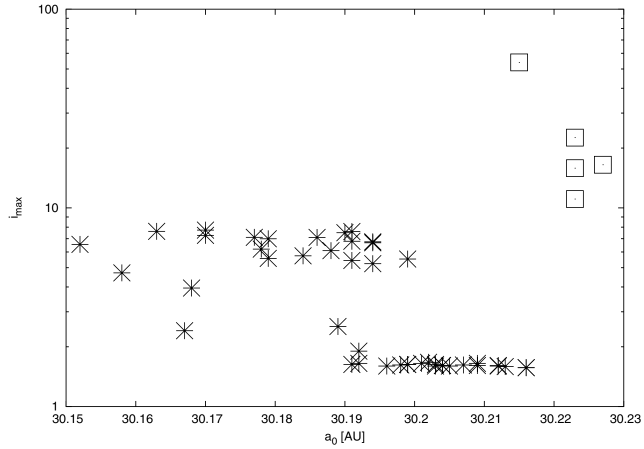


**Figure 6.** The maximum observed inclination versus initial semimajor axis. Note the abrupt end of the $\nu_{18}$ resonance for $a_0 \sim 30.195$ au.

Generally, those trajectories which remained deep inside the $\nu_{18}$ resonance for the whole 5 Gyr have the longest e-folding time, whereas those orbits for which the argument $\Omega$–$\Omega_8$ switches (irregularly) between libration and circulation, the e-folding time is much shorter.

## 4 SUMMARY AND CONCLUSIONS

The dynamics of the Neptune Trojan 2001 QR322 is diverse. The uncertainty in its observed motion leaves room for speculation about its long-term evolution; as a result, we integrated more than one hundred possible orbits. The main results are:

(i) The libration of these orbits around the $L_4$ point has an amplitude of some 40°–75° and corresponding periods from 8900 to 9300 yr. At the moment, the semimajor axis of this Neptune Trojan is larger than that of Neptune itself, so that the two bodies approach each other.

(ii) The motion is subject to one secular resonance: the $\nu_{18}$ resonance. Based on our results, there is an approximately equal probability for the true orbit to lie inside or outside the $\nu_{18}$ resonance. For the orbits initially caught in the resonance, the argument $\Omega$–$\Omega_8$ switches irregularly between libration and circulation.

(iii) The motion is chaotic and the Lyapunov time is somewhere between 1.7 and 20 Myr.

(iv) The $\nu_{18}$ resonance may increase the inclination by as much as 7°.

(v) The main conclusion is that it is quite likely that the Trojan is a *primordial* object because, to high probability, its motion has been stable for the last 5 Gyr.

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