A phase in a material is a region that differ in its microstructure and or composition from another region

- homogeneous in crystal structure and atomic arrangement
- have same chemical and physical properties throughout
- have a definite interface and able to be mechanically separated from its surroundings

Additional resources: Callister, chapter 9 and 10
Phase diagram and “degrees of freedom”

A **phase diagrams** is a type of graph used to show the *equilibrium* conditions between the thermodynamically-distinct phases; or to show what phases are present in the material system at various $T$, $p$, and compositions.

- “equilibrium” is important: phase diagrams are determined by using slow cooling conditions ⇒ no information about kinetics

**Degree of freedom (or variance) $F$** is *the number* of variables ($T$, $p$, and composition) that can be changed independently without changing the phases of the system.

![Phase diagram of CO$_2$](image)
8.1 Phase Diagram of Water

- **Field** – 1 phase
- **Line** – phase coexistence, 2 phases
- **Triple point** – 3 phases

3 phases: solid, liquid, vapour

**Triple point:**
4.579 Torr
(~603 Pa),
0.0098°C
8.2 Gibbs Phase Rule

Gibbs' phase rule describes the possible number of **degrees of freedom** \((F)\) in a **closed system** at equilibrium, in terms of the number of separate **phases** \((P)\) and the number of **chemical components** \((C)\) in the system (derived from thermodynamic principles by Josiah W. Gibbs in the 1870s).

\[
F + P = C + 2
\]

- **F** is the number of degrees of freedom or variance
- **P** is the number of phases
- **C** is the number of components

**Component** is the minimum number of species necessary to define the composition of the system.

- \(\text{H}_2\text{O} \quad C=1\)
  - (i) \(P=1, F=2;\)
  - (ii) \(P=2, F=1;\)
  - (iii) \(P=3, F=0\)

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**PHASE DIAGRAM for H\textsubscript{2}O**

- **Melting**
- **Freezing**
- **Boiling**
- **Condensing**
- **Sublimating**

**Temperature (°C)**

**Pressure (Pa)**
8.3 How to construct phase diagrams? - Cooling curves

Cooling curves:
- used to determine phase transition temperature
- record T of material vs time, as it cools from its molten state through solidification and finally to RT (at a constant pressure!!!)

The cooling curve of a pure metal

**BC**: plateau or region of thermal arrest; in this region material is in the form of solid and liquid phases

**CD**: solidification is completed, T drops
Cooling curve for pure iron @ 1atm

As $T \downarrow$: melted iron (liquid) $\Rightarrow$ $bcc$ Fe, $\delta$ (solid) $\Rightarrow$ $fcc$ Fe, $\gamma$ (solid) $\Rightarrow$ $bcc$ Fe, $\alpha$ (RT)
8.4 Binary systems (C = 2)

\[ F + P = C + 2 = 4 \implies F = 4 - P \]

Degrees of freedom (F):
p, T, composition

At \( p = \text{const} \) (or \( T=\text{const} \))

\[ F = 3 - P \]

1. Two components are completely \textbf{mixable} in liquid and solid phase (form a solid state solution), and don’t react chemically
2. Two components (A and B) can form \textbf{stable compounds} or alloys (for example: A, \( A_2B \), \( A_3B \), B)
Binary Isomorphous Alloy System (C=2)

**Isomorphous:** Two elements are completely soluble in each other in solid and liquid state; substitutional solid state solution can be formed; single type of crystal str. exist

**Reminder: Hume-Rothery rules:** (1) atoms have similar radii; (2) both pure materials have same crystal structure; (3) similar electronegativity (otherwise may form a compound instead); (4) solute should have higher valence

Example: Cu-Ni phase diagram (only for slow cooling conditions)

**Liquidus line:** the line connecting Ts at which liquid starts to solidify under equilibrium conditions

**Solidus:** the temperature at which the last of the liquid phase solidifies

Between liquidus and solidus: $P = 2$
53 wt% Ni – 47 wt% Cu at 1300°C

- contains both liquid and solid phases \(\Rightarrow\) neither of these phases can have average composition 53 wt% Ni – 47 wt% Cu

- draw a tie line at 1300°C \(\Rightarrow\) from the graph: composition of liquid phase \(w_L = 45\%\) and solid phase \(w_S = 58\%\) at 1300°C
8.5 The Lever Rule

The weight percentages of the phases in any 2 phase region can be calculated by using the **lever rule**

Consider the binary equilibrium phase diagram of elements A and B that are completely soluble in each other.

Let $x$ be the alloy composition of interest, its mass fraction of B (in A) is $C_o$

Let $T$ be the temperature of interest $\Rightarrow$ at $T$ alloy $x$ consists of a mixture of liquid (with $C_L$ - mass fraction of B in liquid) and solid ($C_s$ - mass fraction of B in solid phase)
Lever Rule (cont.)
Q.: A Cu-Ni alloy contains 47 wt % Cu and 53% of Ni and is at 1300°C. Use Fig.8.5 and answer the following:

A. What is the weight percent of Cu in the liquid and solid phases at this temperature?
B. What weight percent of this alloy is liquid and what weight percent is solid?
8.6 Nonequilibrium Solidification of Alloys

Atomic diffusion is slow in solid state; as-cast microstructures show “core structures” caused by regions of different chemical composition.
Nonequilibrium Solidus

Solidification of a 70% Ni-30%Cu alloy

Fig. 8.9, Smith

Fig. 8.10, Smith

- each core structure will have composition gradient \( \alpha_1-\alpha_7 \)

- additional **homogenization** step is often required (annealing \(<T_7\))

Chapter 8
8.7 Binary Eutectic Alloy System

- Components has **limited** solid solubility in each other
- Example: cooling 60%Pb – 40%Sn system

\[ \text{Liquid} \xrightarrow{\text{eutectic } T} a \text{ solid solution} + b \text{ solid solution} \]

This eutectic reaction is called an **invariant** reaction ⇒ occurs under equilibrium conditions at specific T and alloy composition

\[ F=0 \]

at **eutectic** point
Solubility Limit: Water-Sugar

- Changing $T$ can change # of phases: path $A$ to $B$.
- Changing $C_o$ can change # of phases: path $B$ to $D$

Adapted from Callister
Binary Eutectic Alloy System

Figure 8.13, Smith
Q: A lead-tin (Pb – Sn) alloy contains 64 wt % proeutectic (α) and 36% eutectic α+β at 183°C – ΔT. Using Figure 8.13, calculate the average composition of this alloy.
8.8 Binary Peritectic Alloy System

The melting points of the two components are quite different.

A liquid phase reacts with the solid phase to form a new and different solid phase.

\[
\text{Liquid + } \alpha \rightarrow \beta 
\]
Binary Peritectic Alloy System (cont.)

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[Diagram showing a phase diagram with various lines, points, and regions labeled with temperatures and compositions. The diagram includes phases labeled as $\alpha$, $\beta$, $\alpha + \beta$, $L + \alpha$, and $L + \beta$.]

Temperature (°C)

1800
1600
1400
1200
1000
800
600
400
200
0

Weight percent silver

10.5
1186°C
42.4
48
66.3
77

100% Pt

0
10
20
30
40
50
60
70
80
90
100%

Ag

Chapter 8
8.9 Binary monotectic systems

Monotectic reaction: a liquid phase transforms into a solid phase and another liquid phase

\[ L_1 \rightarrow \alpha + L_2 \]
8.10 Invariant Reactions

To summarize:
5 invariant reactions (F = 0)

1. Eutectic  
   Liquid → α + β

2. Eutectoid  
   α → β + γ

3. Peritectic  
   Liquid + α → β

4. Peritectoid  
   α + β → γ

5. Monotectic  
   L₁ → α + L₂

The eutectic and eutectoid reactions are similar in that they both involve the decomposition of a single phase into two solid phases. The –oid suffix indicates that a solid, rather than liquid, phase is decomposing.
8.11 Phase Diagrams with Intermediate Phases and Compounds

**Terminal phase**: a solid solution of one component in another for which one boundary of the phase field is a pure component

**Intermediate phase**: a phase whose composition range is between those of terminal phases
Ti-Si-O system

- Experiment (700-1000°C)
  \[ \text{Ti} + \text{SiO}_2 \rightarrow \text{Ti}_5\text{Si}_3 \text{ and TiO}_y \]

- At equilibrium the system will be in \( \text{TiSi}_x - \text{TiO}_y - \text{SiO}_2 \) three phase region (from calculations)

- \( \text{Ti}_5\text{Si}_3 - \text{TiO} - \text{SiO}_2 \) three phase region determined experimentally and remaining tie lines can be inferred
8.12 Ternary Phase Diagram

\[ F + P = C + 2 \]

For \( p = 1 \text{atm}, \ T = \text{const (isoterms)} \)

Cr-Fe-Ni alloy stainless steel
Three and four component system

AB + AC = 2A + BC

\[ \Delta G = (2G_A + G_{BC}) - (G_{AB} + G_{AC}) \]

If \( \Delta G < 0 \), there is a tie line between A and BC.
The remaining tie lines cannot cross.

AB + AC + AD = 3A + BCD

\[ \Delta G = (3G_A + G_{BCD}) - (G_{AB} + G_{AC} + G_{AD}) \]

- Two phase equilibrium is represented by a tie line.
- If \( \Delta G < 0 \), there is a tie line between A and BCD;
- otherwise plane connects AB-AC-AD.
The Ti-Si-N-O quaternary phase diagram

- Entire phase diagram can be calculated by taking into account all possible combinations of reactions and products
- 4 ternary diagrams of Ti-Si-N, Ti-N-O, Ti-Si-O and Si-N-O were evaluated
- additional quaternary tie lines from TiN to SiO$_2$ and Si$_2$N$_2$O
- stable metallization bilayer of TiN and TiSi$_2$ in contact with SiO$_2$
