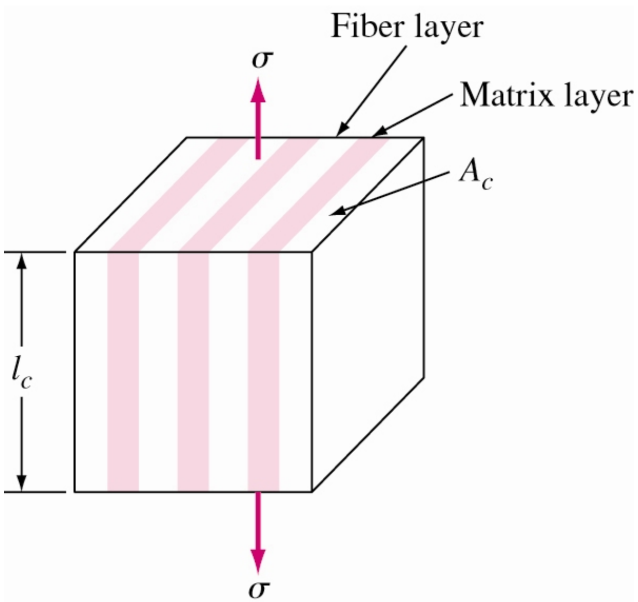
Young's Modulus for Composites for Isostrain Conditions

Derive an equation relating the elastic modulus of a layered composite of unidirectional fibers and a plastic matrix that is loaded under isostrain conditions.



The load on the composite is equal to the sum of the loads on the fiber layers and the matrix layers or $F_C = F_F + F_M$. Since $F = \sigma A$, we obtain

$$\sigma_C A_C = \sigma_F A_F + \sigma_M A_M$$

But the lengths of the fiber and matrix layers are equal, so the areas can be replaced by the respective volumes:

$$\sigma_C V_C = \sigma_F V_F + \sigma_M V_M$$

For **isostrain** conditions, all strains are equal ($\varepsilon_C = \varepsilon_F = \varepsilon_M$). Thus we can divide each term by its corresponding strain:

$$\frac{\sigma_C V_C}{\varepsilon_C} = \frac{\sigma_F V_F}{\varepsilon_F} + \frac{\sigma_M V_M}{\varepsilon_M}$$

Substituting the modulus of elasticity for σ/ ε,

$$E_C V_C = E_F V_F + E_M V_M \, ; E_C = \frac{E_F V_F + E_M V_M}{V_C}$$